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APPENDIX TO REPLY COMMENTS FILED BY BUSINESS OPTIMIZATION SERVICES

Introduction

This appendix describes methods used to conduct the compliance feasibility analysis presented in the narrative. Calculations presented here were used to determine: whether compliance with the 30 percent institutional cost share recommendation is possible under various likely elasticity scenarios; and for those scenarios indicating compliance, the minimum price increases required. In general, scenario results depend on the assumed demand elasticities as explained in the narrative.

The description is presented in three parts. First, calculations for revenue changes resulting from competitive product price changes are explained. Calculations for cost changes and the contribution impacts resulting from the revenue and cost changes are then presented in the second section. The last section explains procedures used to develop sensitivity analysis results based on the calculations presented in the first two sections.

Because the Postal Service does not publish demand elasticity estimates for its competitive products, certain simplifying assumptions needed to be made for the analysis, as described below. First, cross price effects on demand across USPS products are ignored. Thus, volume responses are assumed to come only from own price changes.[1] Normally, this would mean that price increases required to increase total contribution are overstated because of diversion of a portion of each product's volume loss to other USPS competitive products. However, USPS products are very imperfect substitutes because of shape and performance (speed of delivery) differences and therefore any volume-related substitution among USPS products is likely limited. Most of the USPS volume loss would be diverted to UPS and FEDEX products with the same shape and similar performance targets.[2]

If competitors responded to USPS price increases by raising their own rates, then part of the volume loss would be diverted back to the USPS although total market volume (across the three competitors) would be less because of across the board price increases. Of course, this scenario would be most detrimental to mailers. However, to estimate the impact on contribution from the volume diversion back to the USPS would require knowledge of competitor cross price demand elasticities and of course these data are also unavailable. Thus, the

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1 In economic parlance, there is no shift in a product's demand curve because of price changes in other substitutable products. Changes in volume come only from "riding" up or down a given demand curve because of changes in the corresponding product's rates.

2 For example, volume losses for USPS priority express mail would be mostly diverted to UPS and FEDEX overnight mail, not to USPS priority mail because the next day delivery requirement is a firm constraint for many of these mailings.

analysis assumes that competitors gain volume from USPS price increases but keep rates steady.

Last, calculations for revenue and cost changes assume linear demand and constant marginal costs, respectively. With respect to linear demand, this form of the demand function comports with market realities. It is reasonable to assume that for a high enough product price market demand is zero. A linear demand function passing through an existing data point (volume, price), as assumed in this analysis, gives such a point. The constant marginal cost assumption simplifies the cost calculation at the expense of not fully capturing infra-marginal costs. However, John Panzar has shown that this effect is negligible for partial volume changes, as applies here, and cost elasticity estimates that apply to the USPS cost structure.[3] In any event, despite the necessary simplifications, the analysis does give some idea of the magnitude of possible rate increases that would follow enactment of UPS and FEDEX proposals.

Revenue Effects from Rate Changes

To begin, suppose there are n number of competitive products with each product's demand affected only by own product price. Thus, for

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[3] See "The Role of Costs for Postal Regulation" by John C. Panzar, filed with the Postal Regulatory Commission on September 30, 2014.

any product $i \, \varepsilon \, \{1,2, \dots, n\}$, revenue derived from that product is simply $R_i(p_i) = D_i(p_i)p_i$ where p_i is the product's price and $D_i(p_i)$ is the product's demand function. Then total USPS revenue R_T from the competitive sector can be expressed as the sum of individual product revenues:

$$R_T = \sum_{i=1}^{n} R_i(p_i) = \sum_{i=1}^{n} D_i(p_i)p_i.$$

Thus, R_T is a function of all p_i. For our purposes, we are interested in estimating the change in R_T forthcoming from equal percentage changes in all p_i. This is the same as saying that price levels must always be in the same proportion exhibited by current rates. To show this, define current competitive revenue as $\sum_{i=1}^{n} R_i(p_i)$ where all p_i define current rates. Next, define a proportionality factor $k \geq 0$ affecting all price movements from current levels. Then any change in total revenue ΔR_T yielded by equal percent price changes must be according to:

$$\Delta R_T = \sum_{i=1}^{n} R_i(p_i k) - \sum_{i=1}^{n} R_i(p_i)$$

where $p_i k$ is the new rate for some product i and $\sum_{i=1}^{n} R_i(p_i k)$ is the new total revenue. Note that current rates are implicitly defined by $k = 1$ and therefore the k factor can be dropped in showing current revenue.

If the functional form and related parameter values for each product's revenue were known, then ΔR_T could be calculated by subtituting values for p_i and $p_i k$ directly into expressions. However, without that information, an approximation for ΔR_T can still be calculated through a Taylor series second order expansion. The revenue change resulting from a proportional change in all rates can be approximated by the following Taylor series estimate:

$$\Delta R_T \approx \sum_{i=1}^{n} \frac{dR_i}{dk} \Delta k + \frac{1}{2} \sum_{i=1}^{n} \frac{d^2 R_i}{(dk)^2} \Delta k^2.$$

The changes in k and the indicated first and second order marginal effects $\frac{dR_i}{dk}$ and $\frac{d^2 R_i}{(dk)^2}$ are estimated at the existing data points. Thus, the revenue changes are calculated from the same points, as necessary.

Since Δk is just the difference between k and one (at current rates), the last can be re-expressed as:

$$\Delta R_T \approx \sum_{i=1}^{n} \frac{dR_i}{dk} (k-1) + \frac{1}{2} \sum_{i=1}^{n} \frac{d^2 R_i}{(dk)^2} (k-1)^2.$$

This calculation can be converted more usefully into a form using demand elasticities as follows. First, note that the chain rule of calculus can be applied to the product i revenue function $R_i(p_i k) = D_i(p_i k) p_i k$ to calculate marginal revenue with respect to k as $\frac{dR_i}{dk} = R_i'(p_i k) p_i =$

$D_i(p_i k)p_i + D_i'(p_i k){p_i}^2 k$ where p_i is the current rate and $p_i k$ is any other rate by $k \neq 1$. Differentiating $\frac{dR_i}{dk}$ with respect to k once again yields the following rate of change in marginal revenue $\frac{d^2 R_i}{(dk)^2} = R_i''(p_i k){p_i}^2 = 2D_i'(p_i k){p_i}^2 + D_i''(p_i k){p_i}^3 k$. Recall that the expressions are evaluated at current rates ($k = 1$), so these can be shown as $\frac{dR_i}{dk} = R_i'(p_i)p_i = D_i(p_i)p_i + D_i'(p_i){p_i}^2$ and $\frac{d^2 R_i}{(dk)^2} = R_i''(p_i){p_i}^2 = 2D_i'(p_i){p_i}^2 + D_i''(p_i){p_i}^3$. Finally, with linear demand, $D_i''(p_i) = 0$ and the change in marginal revenue reduces to $\frac{d^2 R_i}{(dk)^2} = R_i''(p_i){p_i}^2 = 2D_i'(p_i){p_i}^2$.

Also with linear demand, the revenue function is quadratic, so conveniently the Taylor estimate for the revenue change is exact. Thus, with the appropriate substitutions from above, the revenue change can be shown as:

$$\Delta R_T = \sum_{i=1}^{n} (D_i(p_i)p_i + D_i'(p_i){p_i}^2)(k-1) + \frac{1}{2}\sum_{i=1}^{n} 2D_i'(p_i){p_i}^2(k-1)^2$$

or

$$\Delta R_T = \sum_{i=1}^{n} (D_i(p_i)p_i + D_i'(p_i){p_i}^2)(k-1) + \sum_{i=1}^{n} D_i'(p_i){p_i}^2(k-1)^2.$$

Then multiplying and dividing both terms on the right by $R_i = D_i(p_i)p_i$ gives the following demand elasticity form for the revenue change:

$$\Delta R_T = \sum_{i=1}^{n} R_i(1 + e_i)(k - 1) + \sum_{i=1}^{n} R_i e_i (k - 1)^2.$$

or

$$\Delta R_T = \sum_{i=1}^{n} R_i[(1 + e_i)(k - 1) + e_i(k - 1)^2].$$

where $e_i = \frac{D_i'(p_i)p}{D_i(p_i)}$. Notice that the revenue change for a particular product i is given by the expression inside the summation sign, $\Delta R_i = R_i[(1 + e_i)(k - 1) + e_i(k - 1)^2]$. The sum of all product revenue changes give the total revenue change as shown.

Cost Effects and Contribution Impacts from Rate Changes

Cost effects from rate changes can also be calculated using a Taylor approximation. Suppose, we categorize u_i as the constant marginal cost for product i. Then total competitive costs can be shown as the following sum of all product i costs:

$$C_T = \sum_{i=1}^{n} C_i(p_i) = \sum_{i=1}^{n} D_i(p_i)u_i.$$

As with revenues, we are interested in estimating the change in C_T forthcoming from equal percentage changes in all p_i according to:

$$\Delta C_T = \sum_{i=1}^{n} D_i(p_i k)u_i - \sum_{i=1}^{n} D_i(p_i)u_i$$

The corresponding Taylor series expansion from the existing data points is:

$$\Delta C_T \approx \sum_{i=1}^{n} D'(p_i)p_i u_i(k-1) + \frac{1}{2}\sum_{i=1}^{n} D''(p_i){p_i}^2 u_i(k-1)^2.$$

Since $D''(p_i) = 0$ with linear demand, the calculation reduces to an exact linear (first order) estimate:

$$\Delta C_T = \sum_{i=1}^{n} D'(p_i)p_i u_i(k-1).$$

With the appropriate substitutions as before, this can be shown in the following demand elasticity form:

$$\Delta C_T = \sum_{i=1}^{n} R_i \frac{u_i}{p_i} e_i(k-1),$$

where $R_i \frac{u_i}{p_i} e_i(k-1)$ is the cost change for any product i. The sum of these terms gives the total cost change.

Finally, define π_T as the total competitive product contribution. Then the change in total contribution is simply the difference between the revenue and cost changes. Substitution from above yields:

$$\Delta\pi_T = \sum_{i=1}^{n} R_i[(1 + e_i)(k - 1) + e_i(k - 1)^2] - \sum_{i=1}^{n} R_i \frac{u_i}{p_i} e_i(k - 1).$$

Collecting terms, the total contribution change can also be shown as:

$$\Delta\pi_T = \sum_{i=1}^{n} R_i \left[\left(1 + e_i \left(1 - \frac{u_i}{p_i} \right) \right) (k - 1) + e_i(k - 1)^2 \right]$$

where $\Delta\pi_i = R_i \left[\left(1 + e_i \left(1 - \frac{u_i}{p_i} \right) \right) (k - 1) + e_i(k - 1)^2 \right]$ is the change in the product i contribution. The sum of contribution changes from all products gives the total change, as shown.

Structure of the Sensitivity Analysis

The sensitivity analysis used the calculation for the contribution change to estimate by demand elasticity scenario: the maximum contributions possible from competitive products; the related percent price changes; and in cases where compliance with the 30 percent institutional cost share is feasible, the minimum percent price change yielding compliance. Procedures used for these determinations involved several steps, as described below.

The first step involved dividing letter and package products into two markets, as described in the comments, and then assigning the same elasticity to each product within a market by scenario. Thus, for a particular scenario, a particular letter market elasticity e_L was assigned to all letter products and a particular package market elasticity e_P was assigned to all package products. Thus assuming m letter products and $n - m$ package products, the change in total contribution becomes:

$$\Delta\pi_T = \sum_{i=1}^{n} R_i \left[\left(1 + e_i \left(1 - \frac{u_i}{p_i} \right) \right) (k-1) + e_i (k-1)^2 \right]$$

where $e_i = e_L$ for all i from 1 to m and $e_i = e_P$ for all i from $m + 1$ to n.

Next, after assigning the scenario-specific elasticities, the contribution maximizing percent price change for each scenario was calculated by determining the marginal effect of k on $\Delta\pi_T$ and setting the result equal to zero. This yields:

$$\frac{d(\Delta\pi_T)}{dk} = \sum_{i=1}^{n} R_i \left(1 + e_i \left(1 - \frac{u_i}{p_i} \right) \right) + 2(k-1) \sum_{i=1}^{n} R_i\, e_i = 0,$$

and solving for $k - 1$ gives the optimal solution:

$$k - 1 = \frac{\sum_{i=1}^{n} R_i \left(1 + e_i \left(1 - \frac{u_i}{p_i} \right) \right)}{-2 \sum_{i=1}^{n} R_i\, e_i}.$$

Then plugging the solution value for $k-1$ back into the calculation for $\Delta\pi_T$ gives the maximum $\Delta\pi_T$ for that scenario. Compliance is feasible only if the maximum $\Delta\pi_T$ is equal to or greater than the change required for compliance. This calculation is repeated for each scenario to determine the overall extent of compliance feasibility.

For those scenarios indicating compliance, the quadratic formula was applied to calculate the minimum percent price changes required for compliance. The formula states that for any quadratic taking the general form $ax^2 + bx + c = 0$, there are real valued solutions for x given by $x_1 = -\frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}$ and $x_2 = -\frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}$ when $b^2 - 4ac \geq 0$. Notice that the total contribution increase can be restated in quadratic form according to:

$$(k-1)^2 \sum_{i=1}^{n} R_i\, e_i + (k-1) \sum_{i=1}^{n} R_i \left(1 + e_i \left(1 - \frac{u_i}{p_i}\right)\right) - \Delta\pi_T = 0.$$

where $x = (k-1)$, $a = \sum_{i=1}^{n} R_i\, e_i$, $b = \sum_{i=1}^{n} R_i \left(1 + e_i \left(1 - \frac{u_i}{p_i}\right)\right)$ and $c = -\Delta\pi_T$. Thus, by letting $\Delta\pi_T$ equal the contribution increase required to meet the 30 percent institutional cost share recommendation, the formula can be applied to calculate $(k-1)$.

As indicated, the quadratic formula normally yields a high and low value for the solution variable, except where $b^2 - 4ac = 0$ in which case there is only one solution. However, we are only interested in the minimum value for the price increase that yields compliance. Thus, for our purposes, we can restate the solution compactly as $k-1=-\frac{b}{2a}+\frac{\sqrt{b^2+4a\Delta\pi_T}}{2a}$ where $a=\sum_{i=1}^{n}R_i\,e_i$, $b=\sum_{i=1}^{n}R_i\left(1+e_i\left(1-\frac{u_i}{p_i}\right)\right)$ to yield that value.

Several points are worth noting regarding this solution. Notice that $-\frac{b}{2a}$ is just the contribution maximizing value for $k-1$, as indicated before. Thus if $b^2+4a\Delta\pi_T=0$, compliance requires contribution maximization. Otherwise by $b^2+4a\Delta\pi_T>0$, $k-1$ is less than $=-\frac{b}{2a}$, indicating that the contribution required for compliance is less than the maximum. Second, because $b^2+4a\Delta\pi_T=0$ yields the contribution maximizing $k-1$, then $\pi_{TMAX}=\pi_T=-b^2/4a$ must be the contribution change yielding that maximum.[4] Substituting in $b^2+4a\Delta\pi_T$ and manipulating then gives $b^2+4a\Delta\pi_T=-4a(\Delta\pi_{TMAX}-\Delta\pi_T)$. Therefore $k-1=-\frac{b}{2a}+\frac{\sqrt{b^2+4a\Delta\pi_T}}{2a}$ can be restated as:

$$k-1=-\frac{b}{2a}+\frac{\sqrt{-4a(\Delta\pi_{TMAX}-\Delta\pi_T)}}{2a}$$

4 This can be confirmed by substituting $-\frac{b}{2a}$ for $k-1$ in $\Delta\pi_T=a(k-1)^2+b(k-1)$.

or by further manipulation:

$$k - 1 = -\frac{b}{2a} - \sqrt{\frac{\Delta\pi_T - \Delta\pi_{TMAX}}{a}}$$

This last form for the calculation was performed for all demand elasticity scenarios indicating compliance. Note that the prior step determines compliance feasibility ($\Delta\pi_{TMAX} \geq \Delta\pi_T$) and since $a = \sum_{i=1}^{n} R_i\, e_i < 0$, then $\frac{\Delta\pi_T - \Delta\pi_{TMAX}}{a} \geq 0$. Therefore $k - 1 \leq -\frac{b}{2a}$. This confirms that the percent price increases yielding compliance can be no higher than the corresponding increases yielding maximum contributions.