A Branching AIDS Model for Estimating U.S. Postal Price Elasticities

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1. INTRODUCTION

In this paper we describe and apply a method for econometrically estimating a series of complete matrices of price elasticities for U.S. Postal Service (USPS) domestic mail. The matrices correspond to increasingly detailed disaggregations of mail by class, by rate category and by shape.

We begin by econometrically fitting a conventional demand equation, the "trunk" equation, to explain aggregate expenditures for domestic mail services. Next, we fit a branching sequence of share equations based upon the Almost Ideal Demand System (AIDS) model originally developed by Deaton and Muellbauer (1980). In our model, the share equations at the branching points describe the division of postal revenues among mail classes, then by rate categories, and, finally, by shapes.

Our application of this branching AIDS model to postal demand adapts and extends a similar application by Hausman *et al* (1994). Hausman and Leonard (2005) recommend this approach generally as an effective method for fitting a flexible demand model for competitive products. Our results demonstrate that the branching AIDS model works well for USPS domestic mail. The AIDS equations at each branching point provide a good fit of revenue shares to expenditures, prices and other explanatory variables. The elasticities derived from the estimated equations conform well to both basic neoclassical demand theory and the USPS elasticity results expected from conventional demand models.

The modeling results described in this paper demonstrate that modern econometric methods are capable of producing complete matrices of postal price elasticities at a high level of detail and accuracy. The matrices derived from the fitted model show that own-price elasticities of demand are related to the level of aggregation of mail and tend to become larger (in absolute value) as mail categories are disaggregated. The estimates also show that an own-price elasticity drawn from a conventional econometric model that omits cross-price effects is roughly equivalent to the sum of the true own-price elasticity and all the omitted cross-price elasticities. This means that conventional demand models should be adequate for forecasting postal demands when domestic postal rates move in unison.[1] However, it is equally apparent from our estimates that there are many statistically significant cross-price elasticities of demand among U.S. domestic mail services.

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Following this Introduction (Section 1), we describe the overall plan of the model (Section 2). Next, we deal with the trunk equation (Section 3), and its estimation (Section 4). The AIDS share equations are described (Section 5), as well as the method for fitting them (Section 6), followed by the results for the share equations that divide total domestic revenue by major class (Section 7). We also show how to employ the equation fits sequentially to derive the matrices of price elasticities (Section 8). Table 3 contains estimates of the matrix of class-level elasticities; Appendix holds the matrix of estimates at the rate-category level. We also discuss the variance-covariance matrix (Section 9) and the variances of the elasticities (Section 10). We next provide an analysis and summary of the overall properties of the elasticity estimates (Section 11). The paper ends with a brief discussion of data issues that confront further research (Section 12) and the conclusion (Section 13).

2. THE PLAN OF THE MODEL

The key to our model is a structuring of U.S. domestic mail as a tree with branches corresponding to mail categories. The trunk of this tree is the sum of all U.S. domestic mail. The major branches of the tree are domestic classes; secondary branches divide the classes into more refined rate categories by work-sharing or by customer qualifications for preferential rates. The final level of branching divides the majority of rate categories into shapes: letters, cards, flats and parcels.

The application of our branching structure corresponds to a hypothetical budget process for an average mailer. First, the mailer decides on its aggregate budget for domestic mail services. This amount is then divided and subdivided as we proceed up the tree. The basic behavioral assumption of the model is that the budget decision at each branching point requires limited information. For example, determining expenditures by class, requires an average mailer to know only its total expenditure on domestic mail, the price indices for domestic mail classes and other pre-determined conditions represented by exogenous variables. Further up the tree, an average mailer makes its choice at each branching point independently of the choice at other branching points on the same level. Thus, dividing First-Class letter expenditures among work-sharing categories requires no information regarding similar divisions being made on the same level for the other domestic mail classes.

3. THE TRUNK EQUATION

The trunk equation follows the general form of the demand equations fit by Cigno *et al* (2014) for all major categories of U.S. mail. However, we express demand as an aggregate expenditure rather than as an aggregate postal volume (number of pieces). The equation is a restricted trans-log that is a flexible form with respect to postal prices and Internet penetration. All nominal values are deflated to 2009 dollars using the implicit deflator for GDP. The trunk equation is:

$$ln\,(R/H) = \beta_0 + \beta_1 I + \beta_2 ln(P/\bar{P}) + \beta_3 I^2 + \beta_4 ln(P/\bar{P})^2 + \beta_5 I\; ln(P/\bar{P}) + \beta_6 t + \beta_7\, Y_l/H$$

$$+\beta_8\, W/H + \beta_9 \mathrm{D}_{\text{election year}} + \beta_{10-15} \mathrm{D}_{\text{exponential trends}} + u, \text{ where:}$$

$\beta_0, .., \beta_{15}$ are the parameters to be estimated.

The variables of the trunk equation are:

R/H - the postal revenue from domestic mail per Household in a calendar quarter measured in constant-dollar expenditures seasonally adjusted at an annual rate;

I - Internet penetration during the calendar quarter measured as a fraction of U.S. households with broadband service;

$P/\bar{P}$ - the deflated and mean-centered quarterly average revenue per piece of all domestic mail;

t - the autonomous long-term annual rate of change in R/H measured in calendar years starting from the end of FY 2013;

Y_l/H - long-term real GDP per Household defined as a weighted average of the current real quarterly GDP per Household and the previous quarter's Y_l/H.

W/H - real net Worth per Household.

$D_{election\ year}$ - a dummy variable that is set to one for the third and fourth quarters of election years and to zero, otherwise.

$D_{exponential\ trends}$ - exponential trends that are included to represent the effects over time of the introduction of new services, the impact of the 9/11 attack and anthrax mailings.[2]

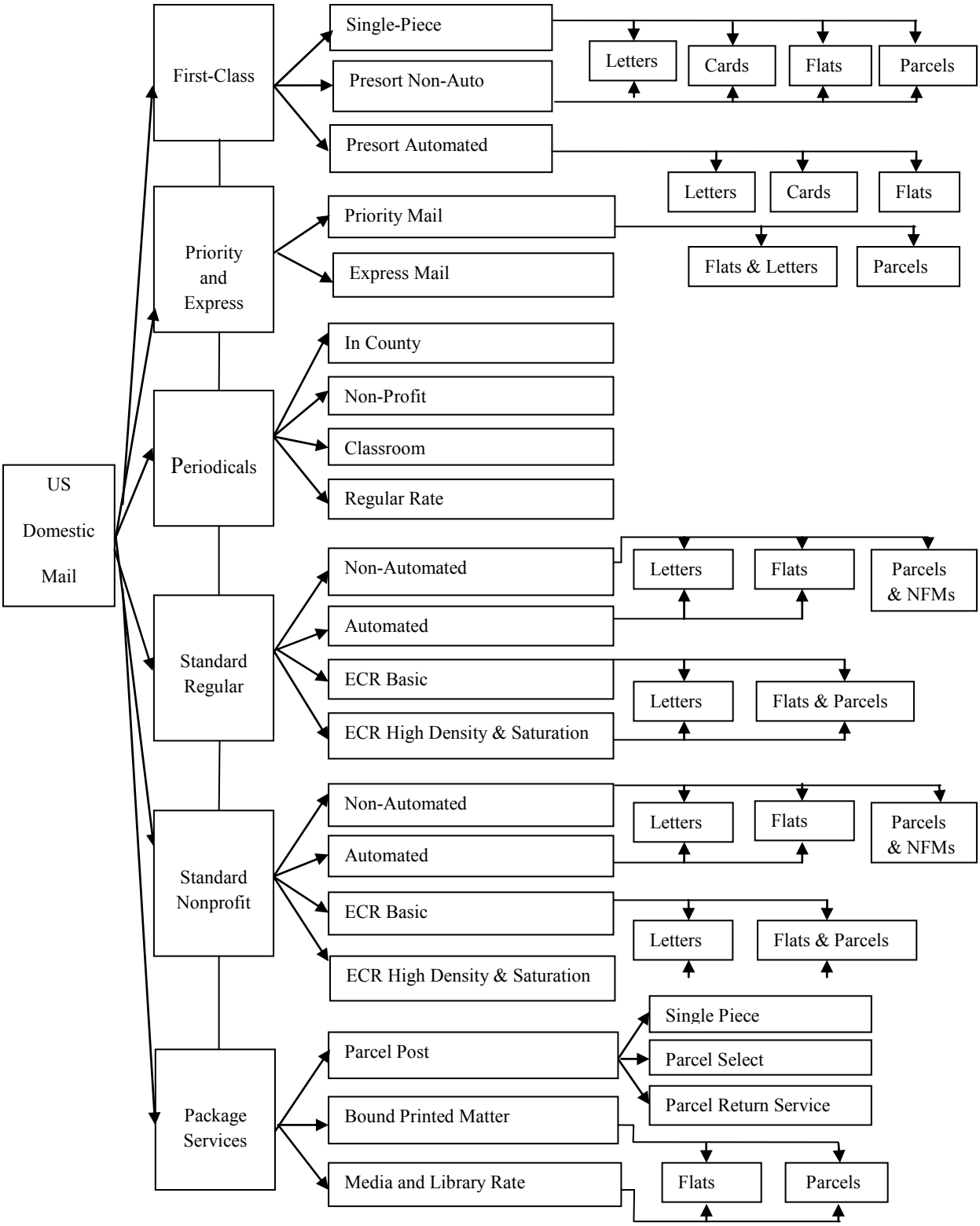
u - the equation error, which is assumed to obey a linear homogenous auto-regressive process with four quarterly lags (AR-4).[3]

The own-price elasticity of the aggregate demand for domestic mail services, ε, is:

$$\varepsilon = -1 + \frac{\partial ln(R/H)}{\partial ln(P/\bar{P})} = -1 + \beta_2 + 2\beta_4 ln(P/\bar{P}) + \beta_5 I.$$

Figure 1 presents the branching tree lying on its side. The tree closely follows the product definitions and aggregations currently used in the quarterly Revenue, Pieces and Weight (RPW) reports that the USPS files with the PRC.

Figure 1: Tree Branching Structure for U.S. Domestic Mail



4. FITTING THE TRUNK EQUATION

The trunk equation has been fit using methods described in detail in Pearsall (2005 and 2011) and Cigno *et al* (2013a) to deal with several complications presented by the form of the equation. These are, first, the AR-4 process of the equation error, second, the rate of adaptation for the exponential trends, and, third, the possibility that the estimates may imply a positive price elasticity within the normal range of postal prices and Internet penetration. Briefly, the AR-4 process is handled by performing an initial least-squares fit of the trunk equation, deriving an estimate of the parameters of the AR-4 process from the residuals, transforming the data so that the transformed errors are serially uncorrelated, and refitting the trunk equation to the transformed data. We have embedded this estimation sequence within a simple search algorithm designed to find the rate of adaptation that maximizes the combined likelihood of the residual equation errors. Confidence intervals for the estimates are determined numerically based on a large-sample Chi-square test. Finally, we have found that when the price elasticity of the aggregate demand for domestic mail services is calculated for the sample extremes of the real price index and for $I = 0$ or $I = 1$ it occasionally is positive. When this happens the equation is re-estimated using mixed Generalized Least Squares (GLS) as described by Cigno *et al* (2013a). This method adjusts the coefficient estimates so that the calculated price elasticities are all non-positive and accomplishes the changes without altering the OLS variance-covariance matrix of the elasticities.

We face an additional complication by using average revenue per piece as the price index for all domestic mail. We calculate average revenue per piece by dividing seasonally adjusted total domestic revenue by seasonally adjusted number of pieces. This calculation eliminates seasonal fluctuations but introduces the equation error u into the measurement of the price index $P/\bar{P}$. The price index is likely to be endogenous for other reasons as well. Average revenue per piece reflects postal customers' collective responses to a complex postal tariff and is an endogenous measure of responses to the tariff.

If we fit the trunk equation by using ordinary methods, the resulting estimates of its parameters will be inconsistent due to the presence of the error in $P/\bar{P}$. Cigno *et al* (2014) avoid this problem by employing a fixed-weight index (FWI) price, rather than average revenue per piece, to fit their demand equations. However, this solution is problematic when the dependent variable is revenue per household or business because average revenue per piece is implicitly present on the left-hand side of the trunk equation as well as explicitly on the right-hand side. The formula for calculating the own-price elasticity from a revenue equation presumes that the definition of price is the same on both sides of the equation.

A practical solution to the endogenous nature of $P/\bar{P}$ is to construct an instrumental variable that is correlated with $P/\bar{P}$ but uncorrelated with the error in its measurement. We do this by fitting a reduced form equation for average revenue per piece using an FWI price. The reduced form equation is the mirror image of the trunk demand equation:

$$ln(P/\bar{P}) = \alpha_0 + \alpha_1 I + \alpha_2 ln(F/\bar{F}) + \alpha_3 I^2 + \alpha_4 ln(F/\bar{F})^2 + \alpha_5 I\ ln(F/\bar{F}) + \alpha_6 t + \alpha_7 Y_l/H$$
$$+\alpha_8 W/H + \alpha_9 \mathrm{D}_{\text{election year}} + \alpha_{10-15} \mathrm{D}_{\text{exponential trends}} + u.$$

$F/\bar{F}$ is the average deflated and mean-centered quarterly FWI price of domestic mail.

The fitted reduced form equation may be evaluated to obtain estimates of $ln(P/\bar{P})$ without the error u for each quarter of the sample. These estimates are the observations for an instrumental variable with the requisite properties. The trunk equation is fit by substituting estimates derived from the reduced form equation for $ln(P/\bar{P})$.

The reduced form equation describes how aggregate revenue per piece responds as mailers' re-optimize their selection of services in response to changes in the postal tariff. The elasticity of revenue per piece with respect to the FWI price is

$$\frac{\partial ln(P/\bar{P})}{\partial ln(F/\bar{F})} = \alpha_2 + 2\alpha_4 ln(F/\bar{F}) + \alpha_5 I.$$

We fit the equation using the same techniques as the trunk equation. The trunk equation and reduced form equation have been fit to quarterly time series spanning 1971 Q3 to 2013 Q4.[4] These estimates are in Table 1.

Both equations show a good fit. The coefficients of the AR-4 processes indicate that the causes of disturbances at the aggregate level do not persist for more than one quarter. Also, postal customers adapt to changes at an estimated annual rate of about 26 percent. When the reduced form equation is used to calculate the elasticity of revenue per piece with respect to the FWI price, the result is significantly less than one.[5]

The fitted trunk demand equation confirms and extends a familiar narrative. The long term trend of U.S. postal demand is downwards due to the gradual encroachment of indirect competitors. During the period from 1976 to about 1995 this trend was obscured by a series of USPS product innovations which successfully stimulated growth in volume and revenue. However, changes made since 1995 have been less successful. The effects of the 9-11 attack and anthrax mailings were minor and temporary. Most recently, the penetration of the Internet has heavily and negatively influenced the demand for domestic mail services. Our estimates also show that changes in the USPS services instituted after the omnibus rate case of 2006 (PRC Docket No. R2006-1) accelerated the decline in household demand for domestic mail services.

In our model, changes in economic activity affect postal volumes and revenues through the variables Y_l/H and W/H. The coefficients of these variables are elasticities of total domestic mail volume with respect to long term GDP and net worth per household. Our estimates show that aggregate postal demand is moderately responsive to changing economic conditions and that the responses tend to lag behind movements in GDP per household.

Table 1: Reduced Form and Trunk Equation Estimates

Total U.S. Domestic Mail

		Reduced Form Ln of Rev per Pc		Trunk Equation Ln of Rev per H'hold	
Explanatory Variable (Regressor)	Effective Date	Estimate	t-value	Estimate	t-value
Intercept		-3.4177	-3.12	-0.3102	-0.34
Internet Penetration (Broadband Connections per Household)		-0.7428	-5.75	-0.4003	-3.50
Mean Centered Domestic Mail FWI Price (Ln of Deflated Fixed-Weight Index / Mean)		0.9116	24.70		
Mean Centered Price of Domestic Mail (Estimated Ln of Deflated Rev per Pc / Mean)				0.7668	16.30
Internet X Internet		0.8355	3.81	0.4063	2.09
Ln FWI Price X Ln FWI Price		2.3388	17.90		
Internet X Ln FWI Price		-0.3279	-0.85		
Ln Rev. per Pc. X Ln Rev. per Pc.				-0.5409	-1.17
Internet X Ln Rev. per Pc.				-0.6495	-1.96
Long Term Annual Trend (Years from the Effective Date)	07/01/2013	-0.0075	-1.58	-0.0237	-6.33
Long Term GDP per Household (Ln of Chained 2009 Dollars/Household)		0.5180	2.35	1.1458	6.01
Real Net Worth per Household (Ln of Chained 2009 Dollars/Household)		-0.0241	-0.64	0.0954	2.67
Election Year Quarters 3 and 4 (Dummy = 1 for Election Yr Qtr 3/4, 0 otherwise)		-0.0018	-0.76	0.0061	2.15
R2006 Rates Installed (Exponential Trend from the Effective Date)	05/14/2007	-0.0279	-0.49	-0.3196	-6.70
9-11 Attack/Anthrax Letters (Reverse Exponential Path from 9/11)	09/11/2001	0.0027	0.21	-0.0134	-0.97
MC95-1 Automation Discounts (Exponential Trend from the Effective Date)	07/01/1996	0.0822	2.99	0.0306	1.38
Standard Saturation Mail Introduced (Exponential Trend from the Effective Date)	02/03/1991	0.0590	2.17	0.0719	3.44
Standard Mail & Car-Rte Presorting Introduced (Exponential Trend from the Effective Date)	03/21/1981	-0.0020	-0.08	0.3650	18.37
3/5-digit 1st Cls Presorting Introduction (Exponential Trend from the Effective Date)	07/06/1976	-0.0159	-0.48	0.1414	5.07
Auto-Regressive Process (AR-4)	Lag Quarter 1	0.5827	7.44	0.3454	4.41
	Lag Quarter 2	-0.1397	-1.55	-0.0604	-0.73
	Lag Quarter 3	0.0176	0.20	-0.0523	-0.63
	Lag Quarter 4	0.0706	0.92	0.0284	0.36
Estimated Annual Rate of Adaptation					
Estimated Rate (Pct. per Year)	26.51%	Adj. R Sqr	0.9778	Adj. R Sqr	0.9996
99 Percent Confidence Interval	+/-15.75%	Std. Error	0.012	Std. Error	0.014
95 Percent Confidence Interval	+/-12.57%	d.f..	150	d.f..	150
90 Percent Confidence Interval	+/-11.00%				

5. THE AIDS SHARE EQUATIONS

The AIDS model is well-suited to predicting the proportion of total revenue derived from each branch for reasons that are described in detail by Hausman and Leonard (2005). Each share equation is a second-order flexible form that does not unreasonably constrain the described demand behavior but can be fit by mostly linear methods. The estimates can also be made to conform to several restrictions derived from neoclassical demand theory. The AIDS equations have the unusual and highly-desirable property that they exactly represent the behavior of an average mailer, even though the equations are ordinarily fit to aggregate data (Deaton and Muellbauer 1980).[6]

At any branching point postal revenue is divided into $i = 1, .., N$ sub-streams according to share equation with the following form:

$$s_i = \alpha_i + \beta_i \ln(Y/P) + \sum_{j=1}^{N} \gamma_{ij} ln(p_j) + \delta_i X_i + u_i, \text{ where:}$$

s_i - the share of postal revenue[7] derived from product i;

Y - the total revenue divided at the branching point;

P - price index for all $j = 1, .., N$ products;

p_j - the price of product j;

X_i - a set of exogenous variables that includes a long-term trend, dummy variables for sudden classification changes and exponential trends to account for changes in market conditions;

$\alpha_i, \beta_i, \gamma_{ij}$ and δ_i - the coefficients to be estimated;

u_i is an additive error that is assumed to obey an AR-4 process as with the error in the trunk equation.[8] The equation for this process is

$u_{it} = \gamma_1 u_{it-1} + \gamma_2 u_{it-2} + \gamma_3 u_{it-3} + \gamma_4 u_{it-4} + \epsilon_{it}$., where ϵ_{it} - is a non-auto-correlated disturbance.[9]

One of the greatest practical advantages of the AIDS model is that several basic results of neoclassical consumer demand theory may be imposed easily as constraints on the coefficients:

Slutsky-Schultz Symmetry: $\gamma_{ij} = \gamma_{ji}$ for $i = 1, .., N$ and $j = 1, .., N$.

Homogeneity of degree zero: $\sum_{j=1}^{N} \gamma_{ij} = 0$ for $i = 1, .., N$.

Adding up: $\sum_{i=1}^{N} \alpha_i = 1$, $\sum_{i=1}^{N} \beta_i = 0$ and $\sum_{i=1}^{N} \delta_i = 0$.

Slutsky-Schultz Symmetry means that the cross-price derivatives of the compensated demands for any two products are equal. This is an outcome of consumer utility maximization subject to a budget constraint. Homogeneity of degree zero means that proportionate changes in all prices and in total revenue, Y, have no effect on the revenue shares. Finally, the Adding Up conditions ensure that the shares always sum to one across all of the products regardless of the values taken by the explanatory variables of the model.

The AIDS model also includes an equation for the log of the price index P:

$$ln(P) = \alpha_0 + \sum_{i=1}^{N} \alpha_i ln(p_i) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} ln(p_i) ln(p_j).$$

If the price index is pre-determined, the share equations of the AIDS model can be fit entirely by linear methods. For this reason the AIDS share equations are often fit using an approximation to the price index that employs fixed weights, w_j, rather than the coefficients

from the share equations. Deaton and Muelbauer (1980) recommend Stone's index: $ln(P) = \sum_{j=1}^{N} w_j ln(p_j)$; the FWI price: $P = \sum_{j=1}^{N} w_j p_j$ is another possibility.[10] We have fit our model by using both alternative indices and have found that the results are quite similar to the estimates we obtain with the AIDS index.

6. FITTING THE AIDS SHARE EQUATIONS

Fitting the share equations for the branching AIDS model presents many of the same complications as fitting the trunk equation, as well as several new ones.

In previous studies (Pearsall 2005 and 2011, Cigno *et al* 2014) rates of adaptation estimated for postal demand models have been found to vary by major class. The rates of adaptation used for fitting our share equations were taken from those estimated for postal classes in Cigno *et al* (2014). We have adjusted these rates proportionately so that their revenue-weighted average matched the annual rate of adaptation of 26.51 percent estimated for all domestic mail. Then for each AIDS share equation we applied the different adaptation rates by class. For example, we used the annual rate of adaptation of 15.06 percent for First-Class mail to calculate long-run real GDP and the exponential trends (whenever they appeared in a share equation for First-Class mail or any of its rate components or shapes).

No attempt was made to adjust the share equation estimates when they yielded positive own-price elasticities. The occasions when such an adjustment might have been desirable proved to be surprisingly few and were encountered almost entirely when the estimates were not statistically significant.

We have employed the same method for dealing with the AR-4 processes in the share equations as with the trunk equations. This was accomplished within the overall scheme for estimating the branching AIDS model. Residuals were computed from a preliminary least-squares fit and used to estimate the parameters of the AR-4 processes. The data was then transformed so that the errors would not be auto-regressive and the equations were refit to the transformed samples. This was accomplished within the overall scheme for estimating the branching AIDS model detailed below.

We have computed the prices appearing in the AIDS share equations by dividing seasonally adjusted revenue for a component of the domestic mail stream by seasonally adjusted volume of the same component. This calculation results in a price series, p_i, that is non-seasonal but endogenous for all of the same reasons given for the aggregate price index P. Our solution to the problem is also the same. For each price appearing in the AIDS equations we have fit a reduced form equation to create an instrument correlated with revenue per piece but not with its measurement error. These equations are all similar log-log forms with explanatory variables that are exogenous with respect to postal demand:

$$ln(p_i) = \alpha_{i0} + \alpha_{i1} I + \alpha_{i2} ln(f_i) + \alpha_{i3} t + \alpha_{i4} Y_l/H + \alpha_{i5} W/H + \alpha_{i6} X_i + u_i.$$

The explanatory variables are:

I, Internet penetration,

f_i - the FWI price of product i,

t- the trend,

Y_l/H- long run real GDP per household,

W/H - real net wealth per household,

X_i - all of the other exogenous variables appearing on the right-hand side of the corresponding share equation

The reduced form equation for $ln(p_i)$ describes how the revenue per piece for a component of the domestic mail stream is affected as mailers respond to changes in the postal tariff and other exogenous variables. The coefficient, α_{i2}, is the elasticity of revenue per piece, p_i with respect to the FWI price, f_i. Again, the error, u_i, is expected to obey an AR-4 process so each reduced form equation is fit by the same three-step method described earlier. $ln(p_i)$ is evaluated from the equation without its error and is used for revenue per piece as required in the fits of the AIDS equations.

Prices are not the only endogenous explanatory variables in the share equations. The total revenue being divided at each branching point, *Y,* is also endogenous within the overall context of the postal demand system, although *Y* is treated as predetermined at each branching point.[11]

We exploit the recursive structure of the model to avoid incorporating an error in *Y* in the share equations at each branching point. This is done by fitting the branching AIDS model in ascending order. First, we fit the trunk equation and evaluate it to obtain an error-free estimate of total revenue from domestic mail. This estimate is used as *Y* in the share equations for the main branches of the tree. These equations divide total revenue among the major classes of mail leaving us with error-free estimates of mail revenue for each major class. Next, the share equations for each secondary branching point are fit using the estimates Y_k for the class *k*. The equations at the secondary branching points further divide postal revenue by work-sharing category or by customer qualifications. This progression is repeated again to fit the share equations that divide the secondary branches by shape.

The properties of the coefficients are imposed on the estimated parameters by arranging the way linear methods are applied to fit the *N* share equations for each branch. Only $N-1$ of the equations are directly fit. The coefficients for the remaining equation are derived from the others using the Homogeneity of degree zero condition and the Adding up condition. The Slutsky-Schultz Symmetry condition and the Homogeneity of degree zero condition make it necessary to estimate only $N-1$ of the coefficients γ_{ij} $(j = 1,..,N)$ appearing in a single share equation. From these conditions we also obtain $\gamma_{iN} = \sum_{j=1}^{N-1} \gamma_{ij}$. Substituting into the *i*-th share equation and collecting terms delivers:

$$s_i = \alpha_i + \beta_i \ln(Y/P) + \sum_{j=1}^{N-1} \gamma_{ij}\left[ln\left(p_j\right) - ln(p_N)\right] + \delta_i X_i + u_i.$$

This is the share equation without γ_{iN}. Slutsky-Schultz Symmetry is imposed by arranging the $N-1$ share equations for a branching point as a single combined share equation with $\gamma_{ij} = \gamma_{ji}$ for $i = 1,..,N-1$ and $j = 1,..,N-1$. This arrangement overlaps the individual share equations so that the coefficients γ_{ij} are all exactly identified.

Given the price index, *P*, the combined share equation for a single branching point may be fit by Generalized Least Squares (GLS). GLS is a more efficient estimator than Ordinary

Least Squares (OLS) in this circumstance because the errors in the combined share equations (after the AR-4 transforms) are not identically and independently distributed (iid). However, in order to apply GLS we first apply OLS to the combined equation and estimate the variance covariance matrices, Ω, from the residuals of the OLS fit. This procedure is commonly called "feasible" GLS. The GLS fit using the OLS estimate of Ω matrices is the last step in the procedure.

If the parameters of the AR-4 processes and the elements of the Ω matrices were known *a priori*, the selection of the branch share equation that is omitted from the combined share equation would be immaterial. As it is, the choice has a small effect on our results because both the AR-4 process for each share equation and the matrix Ω are derived from the residuals of preliminary fits. We have chosen the omitted share equation arbitrarily at each branching point, usually by omitting an equation representing a small share. Although different choices will produce slightly different numerical results, we believe the differences would be too small to materially affect our findings.

A minor detail of the estimation process is the estimation of the scale parameter α_0 appearing in the formula for $ln(P)$. Deaton and Muellbauer (1980) describe α_0 as the aggregate expenditure for a subsistence standard of living when all prices are unity. They recommend that α_0 be determined *a priori.* We have done this by estimating subsistence expenditures from our fitted equations and exploiting the recursive structure of the branching AIDS model. Subsistence expenditures for all domestic mail are obtained by evaluating the trunk equation with subsistence estimates of real GDP and net worth per household, with $P/\bar{P} = 1$, and with all other variables set to their sample averages. This yields α_0 for the estimation of the combined share equation for the main branches. The fitted share equations are then evaluated with the subsistence expenditure, with the prices $p_i = 1$ for $i = 1,..,N$, and with the other variables at their sample averages. These shares are used to divide the aggregate subsistence expenditure among the postal classes providing the estimates of α_0 for the estimation of the share equations for the secondary branching points. The calculation of subsistence expenditures is carried on in this fashion all of the way up the tree. This parameter has little effect on elasticity estimates derived from the combined share equation but is not one of the coefficients of the equation so it must be estimated separately. All of the above depends upon having on hand the observations for the price index P. P is known in advance if we use either Stone's or the FWI price index but is not known in advance if we use the AIDS price index. Then it is calculated from the estimated parameters of the share equations using the formula for $ln(P)$. Our solution to this problem is an iterative process that differs from the scheme proposed by Deaton and Muellbauer (1980) but yields the same coefficient estimates. To initiate the iterative process we calculate Stone's price index using the sample average revenue proportions as fixed weights. The combined share equations are then fit using Stone's index and the resultant coefficient estimates are used to calculate the AIDS price index from the formula for ln(P). Next, the AIDS price index and Stone's index are averaged to obtain a new index P. The new index is used to re-estimate the combined share equation. The new coefficients are used to recalculate again the AIDS price

index. For the next iteration the new AIDS price index and P are averaged to obtain another new index P which is used to re-estimate the combined share equation. The iterations are repeated until the calculated AIDS price index and the P used to fit the combined share equation have converged. In our experience satisfactory convergence typically takes less than ten iterations.

7. ESTIMATES OF THE CLASS-LEVEL SHARE EQUATIONS

The estimates of the coefficients of the class-level AIDS share equations shown in Table 2 are all from a single combined share equation fit for the branching point where our tree divides into major branches - six classes of mail (see Figure 1 in Section 2). This branching point is the first of the 22 branching points for which we have fit a combined AIDS equation. The combined class-level share equation was fit to the quarterly USPS time series for volumes, revenues and prices beginning in 1977 Q1 and ending in 2013 Q4. The combined share equation includes all of the share equations except that for Package Services. The combined equation has 75 explanatory variables and is fit to a combined sample with 720 observations.

Statistically, the AIDS share equations explain the revenue shares by class extremely well as can be seen from the Adjusted R-Squared (.9994) and the Standard Error (0.003). This achievement is not surprising given the large number of explanatory variables in the combined equation. More important is that the estimated coefficients are statistically different from zero. Many of the estimates have absolute t-values that exceed 1.96, the critical value for a two-tail 95 percent test for statistical significance. Altogether this means that the fitted share equations are robust representations of the economic causes affecting postal revenue shares at the class level. The same is true for most of the AIDS share equations we have fit at the category and shape levels.

Table 2: Combined AIDS Share Equations

U.S. Domestic Mail by Major Classes

		First-Class Mail Expenditure Share		Priority & Express Expenditure Share		Periodicals Expenditure Share		Standard Regular Expenditure Share		Standard Nonprofit Expenditure Share		Package Services Expenditure Share	
Explanatory Variable (Regressor)	Effective Date	Estimate	t-value	Estimate	t-value	Estimate	t-value	Estimate	t-value	Estimate	t-value	Estimate	t-value
Intercept (Alpha)		0.6093	6.09	-0.0249	-0.45	0.0442	2.01	0.2816	3.07	0.0242	1.04	0.0656	n.a.
(Alpha Zero = 3.7362)													
Price of First-Class Mail (Gamma)		0.0050	0.13	0.0555	3.07	-0.0473	-6.08	-0.0042	-0.14	-0.0238	-4.07	0.0148	n.a.
(Ln of Rev per Pc)													
Price of Priority & Express Mail (Gamma)		0.0555	3.07	-0.0087	-0.69	-0.0049	-1.04	-0.0263	-2.10	-0.0006	-0.17	-0.0150	n.a.
(Ln of Rev per Pc)													
Price of Periodicals (Gamma)		-0.0473	-6.08	-0.0049	-1.04	0.0235	5.96	0.0221	3.87	0.0033	1.90	0.0034	n.a.
(Ln of Rev per Pc)													
Price of Standard Regular Mail (Gamma)		-0.0042	-0.14	-0.0263	-2.10	0.0221	3.87	-0.0024	-0.08	0.0026	0.50	0.0082	n.a.
(Ln of Rev per Pc)													
Price of Standard Nonprofit Mail (Gamma)		-0.0238	-4.07	-0.0006	-0.17	0.0033	1.90	0.0026	0.50	0.0165	8.96	0.0020	n.a.
(Ln of Rev per Pc)													
Price of Package Services (Gamma)		0.0148	n.a.	-0.0150	n.a.	0.0034	n.a.	0.0082	n.a.	0.0020	n.a.	-0.0134	n.a.
(Ln of Rev per Pc)													
Expenditures for Domestic Mail (Beta)		-0.0205	-0.98	0.0267	2.18	-0.0039	-0.75	-0.0057	-0.30	-0.0019	-0.35	0.0053	n.a.
(Ln Rev per Household / Price Index)													
Long Term Annual Trend	07/01/2013	0.0027	1.29	0.0001	0.07	-0.0014	-3.19	0.0010	0.46	-0.0007	-1.56	-0.0017	n.a.
(Years from the Effective Date)													
Internet Penetration		-0.0838	-3.06	-0.0320	-1.97	0.0093	1.56	0.0824	3.29	0.0096	1.68	0.0147	n.a.
(Broadband Connections per Household)													
R2006 Rates Installed	05/14/2007	-0.0436	-2.46	0.0597	6.01	0.0057	1.56	-0.0469	-2.90	0.0025	0.62	0.0225	n.a.
(Exponential Trend from the Effective Date)													
9-11 Attack/Anthrax Letters	09/11/2001	0.0095	2.39	-0.0035	-1.64	-0.0010	-1.11	-0.0072	-2.10	-0.0010	-1.08	0.0031	n.a.
(Reverse Exponential Path from 9/11)													
MC95-1 Automation Discounts	07/01/1996	-0.0805	-5.95	0.0246	3.07	0.0040	1.39	0.0293	2.30	0.0052	1.85	0.0173	n.a.
(Exponential Trend from the Effective Date)													
Standard Saturation Mail Introduced	02/03/1991	-0.0345	-2.38	0.0269	3.72	0.0006	0.25	-0.0072	-0.53	0.0009	0.32	0.0133	n.a.
(Exponential Trend from the Effective Date)													
Standard Mail & Car-Rte Presorting Introduced	03/21/1981	-0.0068	-0.38	-0.0388	-3.99	-0.0103	-2.76	0.0683	4.46	0.0068	1.73	-0.0191	n.a.
(Exponential Trend from the Effective Date)													
3/5-digit 1st Cls Presorting Introduction	07/06/1976	-0.0381	-1.10	0.0449	2.10	0.0311	4.30	-0.0374	-1.06	0.0094	1.30	-0.0099	n.a.
(Exponential Trend from the Effective Date)													
Election Year Quarters 3 and 4		-0.0003	-0.48	-0.0004	-1.16	0.0003	1.78	0.0000	0.07	0.0004	2.27	0.0000	n.a.
(Dummy = 1 for Election Yr Qtr 3/4, 0 otherwise)													
2012 RPW Report Revisions	01/22/2012	-0.0203	-3.43	0.0064	2.21	0.0005	0.40	-0.0056	-0.97	-0.0027	-2.14	0.0216	n.a.
(Dummy = 1 after Effective Date, 0 before)													
Auto-Regressive Process (AR-4)	Lag Quarter 1	0.7335	8.77	0.4434	5.24	0.3174	3.76	0.7262	8.74	0.4360	5.22		
	Lag Quarter 2	0.0452	0.43	0.1943	2.10	0.1539	1.78	-0.0124	-0.12	0.0592	0.61		
	Lag Quarter 3	-0.0528	-0.51	-0.0110	-0.12	-0.1384	-1.60	0.0164	0.16	0.2903	2.75		
	Lag Quarter 4	-0.1495	-1.79	-0.0321	-0.38	0.0490	0.60	-0.1901	-2.30	-0.3158	-3.41		
Combined Share Equation GLS AR-4	Adj. R Square	0.9994		Rate of Adaptation									
	Std. Error	0.0025		(Pct per Year)									
	d.f.	645		26.51%									

8. THE MATRICES OF PRICE ELASTICITIES

Matrices of price elasticities are derived progressively for each branching level. At each level the equation combines a price elasticity derived for the level below with elasticities taken from the estimated coefficients of the AIDS share equations at the branching level to obtain an element of the matrix of price elasticities.

The progression begins with the own-price elasticity, ε, for all domestic mail with respect to its average revenue per piece, P. This elasticity is combined with three demand elasticities drawn from the estimates of the coefficients of the AIDS share equations for the major branches that divide the mail by classes. These are:

The elasticity of demand for product i with respect to total postal expenditures, Y:

$$\varepsilon_i^Y = (1 + \beta_i / s_i),$$

The elasticity of the AIDS price index, P, with respect to the price, p_j of product j:

$$\varepsilon_P^j = \alpha_j + \sum_{i=1}^{N} \gamma_{ji} ln(p_i),$$ [12] and

The elasticity of demand for product i with respect to the price, p_j of product j:

$$\varepsilon_{ij}^M = -1(i = j) + \left(\gamma_{ij} - \beta_i \varepsilon_P^j\right)/s_i.$$

The latter elasticity, ε_{ij}^M, is a Marshallian elasticity because it is derived under the assumption that the expenditure, Y, is fixed.

The Marshallian elasticity does not capture the entire effect of a change in the price of product j on demand for product i. The complete elasticity of demand for product i with respect to the price of product j may be derived under the assumption that revenue per piece in the trunk equation and the AIDS price index that emerges from the estimates of the share equations are the same.[13] Under this assumption the complete elasticity is obtained by adding to the Marshallian elasticity a term to capture the effect on demand for product i, Q_i, of changes in the price p_j transmitted first to the price index, P, and then on through the effect of P on total postal expenditures, Y:

$$\varepsilon_{ij} = -1(i = j) + \left(\gamma_{ij} - \beta_i \varepsilon_j^P\right)/s_i + \frac{\partial ln(Q_i)}{\partial ln(Y)} \frac{\partial ln(Y)}{\partial ln(P)} \frac{\partial ln(P)}{\partial ln(p_j)},$$

$$\text{where } \frac{\partial ln(Q_i)}{\partial ln(Y)} = \varepsilon_i^Y, \ \frac{\partial ln(Y)}{\partial ln(P)} = 1 + \varepsilon, \text{ and } \frac{\partial ln(P)}{\partial ln(p_j)} = \varepsilon_P^j.$$

We calculate the elements of the matrix of price elasticities for postal volumes disaggregated to the class level by evaluating this formula using the elasticities obtained from fitting the trunk equation and the AIDS share equations for the classes. Substituting elasticities in the formula above we have:

$$\varepsilon_{ij} = \varepsilon_{ij}^M + \varepsilon_i^Y (1 + \varepsilon) \varepsilon_P^j.$$

This is the general version of an equation found in Hausman and Leonard (2005). Technically, it is the formula for a Marshallian demand elasticity because long-term real GDP per household is fixed when we calculate ε. However, postal expenditures are a very small part of an average household's total income. So, ε_{ij}, differs little from the Hicksian (compensated) elasticity. Consequently, we can inspect the estimated elasticities, as we normally would, for compatibility

with neoclassical demand theory ($\varepsilon_{ii} \leq 0$), and to identify substitutes $(\varepsilon_{ij} > 0)$ and complements $(\varepsilon_{ij} < 0)$ when $i \neq j$.

The formula above is sufficient to compute all of the elements of the matrix of price elasticities by major class, i.e. for the major branches of the tree in Figure 1. The information needed to apply the formula is of the same form and origin for every element of the matrix. Specifically, the trunk equation elasticity, ε, is the same for every element, and the other elasticities in the right-hand side of the formula are all derived from the fit of a single combined share equation.

However, the structure of the matrix of price elasticities becomes more complex as we move up the tree. At the next branch level domestic mail for the major classes is subdivided among work-sharing categories or among customer categories. Each class has its own set of share equations combined and fit to postal data disaggregated into work-sharing and customer categories. The matrix of price elasticities by these categories has rectangular blocks corresponding to the elements of the matrix of elasticities by major class:

$$\begin{bmatrix} [\varepsilon_{11}] & \cdots & [\varepsilon_{1N}] \\ \vdots & \ddots & \vdots \\ [\varepsilon_{N1}] & \cdots & [\varepsilon_{NN}] \end{bmatrix}$$

The diagonal blocks are square matrices with elements that apply to mail categories within the same class. The off-diagonal blocks hold cross elasticities between the categories of two different classes. The formulas above are adapted somewhat differently to calculate the elasticities in the diagonal and off-diagonal blocks.

The block $[\varepsilon_{kk}]$ contains all of the own-price and cross-price elasticities for the work-sharing and customer categories within the major mail class *k*. The information that is required to estimate the elements of the block is similar to that used to calculate the elements of the class-level matrix of elasticities. The own-price elasticity for all of the mail in the class is just the *k*-th diagonal element of the class-level matrix ε_{kk}. This elasticity now assumes the previous role of the aggregate own-price elasticity from the trunk equation, ε. The additional information that we need is the three elasticities derived from the coefficients of the AIDS share equations for the major branching point dividing the postal revenues of class *k*. Let Y_k denote total revenue and let P_k be the AIDS price index for class *k*. The three elasticities that we calculate from the fits of the share equations are:

The elasticity of demand for product *i* with respect to the postal expenditures, Y_k:

$$\varepsilon_i^{Y_k} = (1 + \beta_i / s_i),$$

The elasticity of the AIDS price index P_k with respect to the price of product *j*:

$$\varepsilon_{P_k}^{j} = \alpha_j + \sum_{i=1}^{N} \gamma_{ji} ln(p_i), \text{ and}$$

The Marshallian elasticity of demand for product *i* with respect to the price of product *j*:

$$\varepsilon_{ij}^{M_k} = -1(i = j) + \left(\gamma_{ij} - \beta_i \varepsilon_{P_k}^{j}\right) / s_i.$$

Both products *i* and *j* are members of class *k*. The formula for an element, ε_{ij}, of the diagonal block $[\varepsilon_{kk}]$ is:

$$\varepsilon_{ij} = \varepsilon_{ij}^{M_k} + \varepsilon_i^{Y_k}(1 + \varepsilon_{kk})\varepsilon_{P_k}^{j}.$$

An off-diagonal block $[\varepsilon_{kl}]$ holds all of the cross-price elasticities between the products in two different major classes, $k \neq l$. The cross-price elasticity of demand for postal services in class *k* with respect to the class-level price index for class *l* is the element ε_{kl} in the *k*-th row and *l*-th column of the class-level matrix. The division of the total revenue Y_k among the work-sharing and customer categories of class *k* does not require and makes no direct use of the prices that apply to the categories of another class such as class *l*. The elasticities taken from the fits of share equations are:

The elasticity of demand for product *i* with respect to the postal expenditures, Y_k:

$$\varepsilon_i^{Y_k} = (1 + \beta_i/s_i)$$

using the coefficients of the share equations for class *k*.

The elasticity of the AIDS price index P_l with respect to the price of product *j*:

$$\varepsilon_{P_l}^{j} = \alpha_j + \sum_{i=1}^{N} \gamma_{ji} ln(p_i)$$

The elasticity ε_{ij} consists entirely of the effect on demand for product *i*, Q_i, of changes in the price p_j transmitted indirectly through the effect on the postal expenditures for class *k*, Y_k:

$$\varepsilon_{ij} = \frac{\partial ln(Q_i)}{\partial ln(Y_k)} \frac{\partial ln(Y_k)}{\partial ln(P_l)} \frac{\partial ln(P_l)}{\partial ln(p_j)}.$$

After making the substitutions: $\frac{\partial ln(Q_i)}{\partial ln(Y_k)} = \varepsilon_i^{Y_k}$, $\frac{\partial ln(Y_k)}{\partial ln(P_l)} = \varepsilon_{kl}$, and $\frac{\partial ln(P_l)}{\partial ln(p_j)} = \varepsilon_{P_l}^{j}$, we have:

$$\varepsilon_{ij} = \varepsilon_i^{Y_k} \varepsilon_{kl} \varepsilon_{P_l}^{j}.$$

At the highest level of the tree many of the expenditures for work-sharing and customer categories are further subdivided by shape. The mathematics simply repeats with the matrix of price elasticities composed of rectangular blocks that correspond to the elements of the matrix of price elasticities by category.

9. THE VARIANCE-COVARIANCE MATRIX

The variance-covariance matrix of the coefficients of the trunk equation presents no issues if price elasticities calculated from the final OLS fit are non-positive over the historical range of real postal prices and for Internet penetrations in the range $0 \leq I \leq 1$. However, the trunk equation may have been re-estimated by mixed GLS if any of the calculated OLS elasticities are positive numbers. When this happens the variance-covariance matrix of the estimates is also re-estimated as described in Cigno et al (2013a).

The variance-covariance matrix that results from the final GLS fit of a combined share equation at a branching point is the matrix for a conditional distribution of the coefficients appearing in the equation. The distribution is conditional on P, the price index for all N products because the last step in our estimation process treats ln(Y/P) as predetermined. If we follow the

estimation scheme proposed by Deaton and Muellbauer (1980) we can obtain a variance-covariance matrix that is not conditional on P. Unfortunately, the elements of this matrix apply to non-linear combinations of the AIDS model's structural parameters whereas the elements of our conditional matrix apply directly to the coefficients as they appear in the share equations. Therefore, the conditional variances and covariances of linear combinations of the structural parameters may be computed easily from our matrices.

The condition that attaches to the variance-covariance matrix is only a minor handicap when it comes to measurements, such as the own-price elasticity and cross-price elasticities, made at a given location on an aggregate demand function. In order to identify such a location we must calculate the price index P from the equation for ln(P) and use it as though it was known without an error. Therefore, the condition that attaches to our variance-covariance matrix matches an assumption that we would normally make when we use it.

10. APPROXIMATE VARIANCES OF THE ELASTICITIES

Approximate variances may be calculated for the elements of the matrices of elasticities at each level by making several simplifying assumptions.

First, we assume that the postal share, s_i, and the elasticity of the AIDS price index, $\varepsilon_{P_k}^{j}$, are non-stochastic. Basically, this removes from the calculation of the expenditure elasticity, $\varepsilon_i^{Y_k}$, and the Marshallian price elasticity, $\varepsilon_{ij}^{M_k}$ any uncertainty regarding the point on the postal demand curve for which we are making the calculation. Under this assumption the formulas for $\varepsilon_i^{Y_k}$and $\varepsilon_{ij}^{M_k}$ are linear combinations of the coefficients of the combined share equation for the products that divide the total revenue, Y_k, including product i. The variances of these elasticities, denoted $var\left(\varepsilon_i^{Y_k}\right)$ and $var\left(\varepsilon_{ij}^{M_k}\right)$, may readily be derived from the variance-covariance matrix obtained when the combined share equation is fit by the method we have described.

Second, we assume that the Marshallian price elasticity $\varepsilon_{ij}^{M_k}$ is uncorrelated with the other term in the formula for the elasticity elements of the diagonal blocks, $\varepsilon_i^{Y_k}(1+\varepsilon_{kk})\varepsilon_{P_k}^{j}$. Under this assumption we have $var(\varepsilon_{ij}) = var\left(\varepsilon_{ij}^{M_k}\right) + \left[\varepsilon_{P_k}^{j}\right]^2 var\left(\varepsilon_i^{Y_k}(1+\varepsilon_{kk})\right)$ for the elasticities in a diagonal block because $\varepsilon_{P_k}^{j}$ is non-stochastic. For the elements of an off-diagonal block, $\varepsilon_{ij}^{M_k}$ is zero and $\varepsilon_{P_l}^{j}$ is non-stochastic. Therefore, $var(\varepsilon_{ij}) = \left[\varepsilon_{P_l}^{j}\right]^2 var\left(\varepsilon_i^{Y_k}\varepsilon_{kl}\right)$.

Third, we assume that $\varepsilon_i^{Y_k}$ and ε_{kl} are uncorrelated for any two product groups k and l. The elasticities $\varepsilon_i^{Y_k}$ and ε_{kl} are derived from share equations fit at different branch levels so this assumption is equivalent to assuming that the equation errors are independent. The equation for the approximate variances of the elasticities in a diagonal block is:

$$var(\varepsilon_{ij}) = var\left(\varepsilon_{ij}^{M_k}\right) + \left[\varepsilon_{P_k}^{j}\right]^2 \left\{\left[\varepsilon_i^{Y_k}\right]^2 var(\varepsilon_{kk}) + [1+\varepsilon_{kk}]^2 var\left(\varepsilon_i^{Y_k}\right) + var(\varepsilon_{kk}) var\left(\varepsilon_i^{Y_k}\right)\right\},$$

and for an off-diagonal block:

$$var(\varepsilon_{ij}) = \left[\varepsilon_{P_l}^{j}\right]^2 \left\{\left[\varepsilon_i^{Y_k}\right]^2 var(\varepsilon_{kl}) + [\varepsilon_{kl}]^2 var\left(\varepsilon_i^{Y_k}\right) + var(\varepsilon_{kl}) var\left(\varepsilon_i^{Y_k}\right)\right\}.$$

To derive these equations we decompose $var\left(\varepsilon_i^{Y_k}(1+\varepsilon_{kk})\right)$ and $var\left(\varepsilon_i^{Y_k}\varepsilon_{kl}\right)$. Each of these is the variance of the product of two independent random variables. The formula for the variance of the product of two random variables, xy, when x and y are uncorrelated is:

$$var(xy) = [E(x)]^2 var(y) + [E(y)]^2 var(x) + var(x) var(y).$$

11. THE ESTIMATES OF THE ELASTICITY MATRICES

Elasticities derived from our branching AIDS model are not fixed values. They depend somewhat on the prices, for which they are calculated, as well as expenditures and the chosen values for the exogenous variables in the share equations up to the branching level. The estimated elasticities presented in Table 3 and the Appendix, were all derived using sample averages for the year 2013. This is the most recent full year in our time. Therefore, the estimates characterize the most recent demand behavior of U.S. postal customers.

Table 3 provides the estimates for the matrix of the own-price and cross-price elasticities at different branching levels. The more detailed 20 by 20 category-level elasticity matrix is displayed in Appendix.[14] The shape-level matrix is 43 by 43. The elasticity matrices exhibit properties of consumer demand behavior that are close to what we would expect for collections of inter-related domestic mail services. Below are some conclusions derived from the estimates in Table 3 and Appendix.

Table 3: Trunk and Class-Level Elasticities of Demand

Trunk Price Elasticity

	Rev. per Pc.	FWI Price	Elasticity of Rev. per Pc. w/r FWI Price
All Domestic Mail	-0.706	-0.578	0.819
	(-3.21)	(-2.17)	(3.08)

Class-Level Matrix of Revenue per Piece Price Elasticities

	First-Class	Priority/Exp.	Periodicals	Std. Reg.	Std. N-P	Packages	Row Sum Elasticity
First-Class Mail	-0.804	0.122	-0.080	0.079	-0.037	0.043	-0.677
	(-5.54)	(3.32)	(-4.53)	(0.97)	(-2.80)	(2.15)	(-3.91)
Priority and Express Mail	0.524	-1.063	-0.033	-0.170	-0.000	-0.113	-0.856
	(2.42)	(-10.56)	(-0.84)	(-1.36)	(-0.01)	(-2.99)	(-3.09)
Periodicals	-1.477	-0.162	-0.139	0.902	0.131	0.139	-0.606
	(-4.67)	(-0.95)	(-0.97)	(4.12)	(2.10)	(1.89)	(-1.33)
Standard Regular Mail	0.162	-0.091	0.099	-0.925	0.020	0.045	-0.690
	(0.93)	(-1.82)	(4.07)	(-7.17)	(0.93)	(1.34)	(-3.03)
Standard Nonprofit Mail	-0.770	-0.010	0.148	0.200	-0.316	0.097	-0.652
	(-2.60)	(-0.08)	(2.05)	(0.89)	(-4.21)	(1.22)	(-1.56)
Package Services	0.376	-0.237	0.065	0.197	0.040	-1.208	-0.767
	(1.74)	(-3.18)	(1.84)	(1.30)	(1.25)	(-16.27)	(-2.66)

t-values in brackets (.)

- The trunk-level own-price elasticity of all domestic mail with respect to average revenue per piece is estimated as -0.706 and is statistically significant (t-value -3.21). The own-price elasticity of domestic mail with respect to the FWI price index of domestic mail is -0.578.
- The own-price elasticities are always negative. All but one of the class-level own price elasticities are statistically significant at the 95 percent confidence level; 16 out of 20 of the category-level own-price elasticities are negative and significant. Therefore, our estimates comply fully with the most fundamental requirement of demand theory.
- At the highest level of aggregation, the demand for domestic mail services is price inelastic. However, the own-price elasticities tend to become larger in absolute value as we progress up the tree. The own-price elasticities for First-Class, Priority and Express, Standard Regular and Packages are all greater (in magnitude) than -0.706.
- The observed tendency for the own-price elasticities to increase becomes even more pronounced at the category and shape levels where branches define products that are close substitutes for each other. While the own-price elasticity at the class level for Standard Regular mail is -0.925, the estimated own-price elasticities for its categories are -1.160 (for non-automated basic presort), -0.855 (for automated basic presort), -1.764 (for carrier-route basic), and -1.797 (for high-density and saturation carrier-route).

- The cross-price elasticities (the off-diagonal elements of the elasticity matrices) may be either positive (for substitutes) or negative (for complements). However, they are more frequently positive than negative. Periodicals and Standard Regular mail are substitutes for each other at the class level. This is not surprising given that these two classes are broadly competing avenues for advertisers. On the other hand, First-Class mail and Periodicals appear to be complements. This may be partly because much of the advertising in Periodicals invites a mailed response.
- At the category and shape levels the statistically significant cross-price elasticities tend to concentrate within the diagonal blocks corresponding either to classes or categories (depending on branching level). These are the postal products that are most likely to be substitutes. For example, among the three work-sharing categories of First-Class mail there are four significant cross-price elasticities indicating close relationships.

- The sign pattern for cross-price elasticities found in the matrices is fairly symmetric. For example, the cross-price elasticity for Standard Regular mail with respect to the price of Periodicals is 0.099. The corresponding cross-price elasticity for Periodicals with respect to the price of Standard Regular mail is 0.902. Both elasticities are positive indicating that these two categories are substitutes.

- A large negative own-price elasticity of demand usually comes paired with a large positive cross-price elasticity. An extreme example of this pairing is within First-Class mail at the category level. The own-price elasticity for non-automated presort is -15.843. The demand for non-automated presort First-Class mail is highly own-price-elastic. This occurs because the class includes another, and almost identical, work-sharing category - automated First-Class mail. The cross-price elasticity for the non-automated mail with respect to the price of the automated mail is 23.870. Apparently a small change in the price of either category alone is sufficient to induce a large mail flow between them. These flows are a very large percentage of Non-automated presort First-Class because this category had become quite small in volume by 2013.

- Although the own-price elasticities seem to indicate that demand at many category and shape levels is price-elastic, this is mainly because of volume shifts among U.S. domestic mail services when the prices of individual products change. U.S. domestic mail actually remains inelastic at the category and shape level with respect to generalized changes in postal prices. This can be seen by comparing the own-price elasticities with the row sums of the elasticity matrices.
- A substantial proportion of the cross-price elasticities in all of our matrices are statistically significant. In the class-level matrix there are 11 such elements; at the category level there are 115. These cross-price elasticities are effectively zeroed out by conventional demand models.

The row sums of our elasticity matrices are comparable to the own-price elasticities derived from demand models that omit cross-price effects. The row sums in Table 3 and Appendix are roughly within the same ranges as the own-price elasticities shown in Cigno *et al* 2014, Table 4.4. This confirms the same finding in Cigno *et al* 2013b.

Our estimates are demand elasticities with respect to revenue per piece, however, most previous estimates, such as those in Cigno *et al* (2014), are derived from demand equations that have been fit using FWI prices. Revenues per piece and FWI prices are functionally related, and, in principle, one could derive the FWI price elasticities using the formula:

$$\frac{\partial \ln Q}{\partial \ln FWI_i} = \sum_j \frac{\partial \ln Q}{\partial \ln RPP_j} \frac{\partial \ln RPP_j}{\partial \ln FWI_i}$$

Where $\frac{\partial \ln Q}{\partial \ln RPP_j}$ is a demand elasticity with respect to the revenue per piece of product *j* (RPP_j), and $\frac{\partial \ln RPP_j}{\partial \ln FWI_i}$ is the elasticity of RPP_j with respect to the FWI price of product *i* (FWI_i).

In practice this formula can only be applied to the price elasticity of demand for all domestic mail from the trunk equation. The FWI price elasticity corresponding to our revenue per piece elasticity is obtained by multiplying $\frac{\partial \ln Q}{\partial \ln RPP}$ for all domestic mail by $\frac{\partial ln\ RPP}{\partial \ln FWI}$ (taken from the fit of the reduced form equation per Cigno *et al* 2013b). This calculation is shown at the top of Table 3. The own-price elasticity of domestic mail with respect to the FWI price index of domestic mail is -0.578 and is statistically significant (t = -2.17).

To obtain complete matrices of FWI price elasticities we can refit the entire model with the FWI prices substituted for the revenues per piece in the trunk equation and in the AIDS share equations. In effect, the FWI prices are used directly as proxies for revenues per piece rather than to derive instruments by fitting the reduced form equations. The elasticities that result from this approach are elasticities defined as they are in previous models and tend to be smaller in magnitude than the corresponding revenue per piece elasticities presented in Table 3 and Appendix.

12. DATA ISSUES AND FURTHER RESEARCH

A possible better model plan would be to re-order the tree so that the first branches of the trunk represent mail by shape rather than by class. This branching plan corresponds better to a budget process that matches our assumptions because mail services for letters, cards, flats and parcels are not often close substitutes or complements for each other. Unfortunately, the length of the time series data available were not sufficient for the implementation of this branching plan.[15]

While fitting our branching AIDS model to the time series available from the multiple USPS sources, we have attempted to correct a number of problems with the data.

- Until 2003, the RPW data were compiled by postal quarters of unequal length which changed by 1-2 days each year. None of the revenue data, and only some of the earlier volume data, have been recompiled by calendar quarters in the annual filings of the USPS demand model.

- Although the major mail classes have not changed since the Postal Reorganization Act of 1970, there have been many additions, sub-divisions and changes in category definitions. Historical RPW data had not been recompiled to reflect these changes.

- RPW categories with similar unit costs but different demand characteristics have been occasionally aggregated. This has happened to single-piece First-Class cards, to Standard Regular and Standard Nonprofit mail, to all outside-county Periodicals, to several categories of Parcel services, and to U.S. Government mail.

- Since 2008, access to RPW data for competitive classes of mail has been restricted. Several categories of parcels have been reclassified from market dominant to competitive products but the historical data has not been revised to reflect the reclassifications.

The above mentioned transfers and classification changes caused significant obstacles in database development for our model. Therefore, further research with branching AIDS models, along the line taken in this paper, will be difficult until the historical RPW data has been recompiled by shape and by calendar quarters; using a consistent set of product definitions.

It would also be desirable for USPS to seasonally adjust the RPW data. The current practice is to report quarterly data alongside the data for the same period last year (SPLY). The better practice, which is standard with virtually all economic time series regularly collected by U.S. government agencies, is to seasonally adjust if necessary before publication.

13. CONCLUSION

In this paper we have described a flexible and robust method for estimating complete matrices of price elasticities of demand at almost any level of mail product detail for which a suitable sample can be assembled. The method is flexible because it can be applied using a branching scheme tailored to the available data, and because the AIDS share equations at each branching point can be individually specified with respect to the selection of exogenous variables. We have demonstrated that the method is robust by making statistically accurate estimates of own-price and cross-price elasticities for matrices of USPS domestic mail services at several levels of aggregation. These matrices characterize the average recent responses of U.S. households to changes in domestic postal prices. The estimates comply with neoclassical demand theory, generally confirm what is known from previous econometric work, and conform to our expectations regarding the demand behavior of postal customers.

Although conventional demand models allow for forecasting of U.S. postal volumes with fair accuracy, postal rate-setting is more demanding. Setting postal rates requires accurate estimates of cross-price elasticities among postal products, at least, among those that are close substitutes or complements. However, conventional econometric methods do not offer a practical way to estimate them and, typically, ignore cross-price effects by simply omitting the cross-prices from the demand equations. This paper, along with previous papers by Cigno *et al*

(2013b) and Swinand and Hennessy (2014), provides proof that modern econometrics offers several effective ways to obtain complete and consistent matrices of postal price elasticities.

NOTES

1. Real U.S. postal rates usually move in this way, first, because inflation affects all real rates proportionately, second, because changes in the nominal tariff are infrequent, and, third because the changes for market-dominant classes (95 percent of the mail) were capped and tied to increases in the Consumer Price Index by Congress in the Postal Accountability and Enhancement Act (2006).
2. The model includes exponential trends to represent the expansion (or contraction) path of demand following a change in market conditions to which postal customers take time to adapt. The trends start on the date of the event that triggered them and are derived using an estimated common annual rate of adaptation.
3. The AR-4 process is handled by performing an initial least-squares fit of the trunk equation, deriving an estimate of the parameters of the AR-4 process from the residuals, transforming the data so that the transformed errors are serially uncorrelated, and refitting the trunk equation to the transformed data.
4. The source for most of the data is the various USPS filings with the PRC The time series for the aggregate USPS domestic revenues and volumes were recalculated for calendar quarters and seasonally adjusted using the Census X-12 method prior to use.
5. USPS and PRC calculations of revenues from rate changes have always assumed that this elasticity is one. Those calculations also ignore some other factors that affect revenue per piece. For example, revenue per piece increases as GDP per household increases. A possible explanation is a slight increase in the average weight per piece or change in customer shipping preferences (the selection of faster delivery).
6. This property also makes the branching AIDS model a generalized Gorman polar form. As Hausman *et al* (1994) point out the AIDS model is a generalized Gorman polar form and is compatible with an exact two-stage budget process. In order for our entire model to be exactly compatible with a multi-stage budgeting process, each branch point requires a demand model that is a generalized Gorman polar form. We have met this requirement by specifying an AIDS model at each branching point of our tree.
7. Postal shares, s_i, are calculated from seasonally adjusted quarterly revenues and, therefore, are free of purely seasonal effects.
8. The coefficients that apply to Package Services (see Table 2) have been calculated using the Homogeneity of Degree Zero and Adding Up conditions. The coefficient estimates are taken from the final GLS fit of the combined share equation to the AR-4 transformed data.
9. The disturbance ϵ_{it} is serially uncorrelated with a zero mean and stationary variance $\sigma_i^{2.}$ However, the disturbances for the N share equations at a given branching point cannot be independent because of the fact that they must always sum to one. We have assumed that the disturbances for the equations at each branching point have a stationary N by N variance-covariance matrix Ω.
10. Hausman and Leonard (2005) recommend using revenue shares that are averages over the sample period as weights in order to avoid making the price index endogenous.
11. This is the principal operational consequence of using a model that is a generalized Gorman polar form. Total revenue, Y, is predetermined because it is set by a budgeting process that does not depend upon how revenues are sub-divided further up the tree.

12. When Stone's index is used: $\varepsilon_P^j = w_j$; when a FWI price is used: $\varepsilon_P^j = w_j\, p_j / P$.
13. In fact, they are the same in Hausman *et al's* (1994) application of a similar branching model to beer. In that application, the AIDS price indices, calculated from the highest level share equation fits, are used as product prices at the next lowest level. This is not a practical option with U.S. postal prices because of the brevity of the time series that are available for fitting the shape-level AIDS share equations.
14. Both tables include a column vector of sums of the elasticities for each row of the matrix. These row sums are the elasticities of demand with respect to an equi-proportionate change in all postal prices. The t-values for these elasticities have been derived under the assumption that the elasticities composing a row sum are uncorrelated.
15. The USPS began separating postal data by shape only in 2008. As a result, revenues by shape are available only back to 2008 (from the RPW reports filed with the PRC), and partially to 2004 (from other USPS sources). Volumes by shape for the period since 1993, for selected categories can sometimes be gleaned from annual filings supporting the USPS demand model. Our choice of the branching plan for the model was partly driven by the limited availability of USPS data by shape.

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Appendix: Category-Level Matrix of Price Elasticities

Class	Category	First-Class Mail			Priority and Express		Periodicals			
		S-P	Non-Auto	Auto	Priority	Express	In-County	Nonprofit	Classroom	Regular
First-Class Mail	Single-Piece	-0.153	-0.396	-0.206	0.092	0.023	-0.002	-0.012	-0.002	-0.059
		(-0.86)	(-2.63)	(-1.17)	(3.16)	(3.16)	(-4.16)	(-4.16)	(-4.16)	(-4.16)
	Non-Automated	-12.243	-15.843	23.870	0.511	0.128	-0.013	-0.067	-0.012	-0.329
		(-2.55)	(-2.42)	(3.98)	(1.40)	(1.40)	(-1.49)	(-1.49)	(-1.49)	(-1.49)
	Automated	-0.199	0.774	-1.316	0.090	0.023	-0.002	-0.012	-0.002	-0.058
		(-1.05)	(3.95)	(-5.04)	(3.10)	(3.10)	(-4.03)	(-4.03)	(-4.03)	(-4.03)
Priority and Express Mai	Priority	0.312	-0.114	0.345	-1.120	0.019	-0.001	-0.005	-0.001	-0.027
		(2.42)	(-2.42)	(2.42)	(-12.60)	(0.55)	(-0.84)	(-0.84)	(-0.84)	(-0.84)
	Express	0.206	-0.075	0.228	0.573	-1.301	-0.001	-0.004	-0.001	-0.018
		(1.90)	(-1.90)	(1.90)	(2.02)	(-5.09)	(-0.78)	(-0.78)	(-0.78)	(-0.78)
Periodicals	Within-County	-0.907	0.332	-1.000	-0.138	-0.035	-0.102	-0.386	-0.163	0.503
		(-3.09)	(3.09)	(-3.09)	(-0.91)	(-0.91)	(-0.33)	(-1.60)	(-1.16)	(1.21)
	Nonprofit	-0.868	0.317	-0.957	-0.132	-0.033	-0.096	-0.791	0.029	0.716
		(-3.86)	(3.86)	(-3.86)	(-0.93)	(-0.93)	(-1.77)	(-5.81)	(0.96)	(3.30)
	Classroom	-0.423	0.155	-0.466	-0.064	-0.016	-0.569	0.212	-1.021	1.309
		(-1.00)	(1.00)	(-1.00)	(-0.57)	(-0.57)	(-1.29)	(0.50)	(-2.77)	(1.99)
	Regular Rate	-0.850	0.311	-0.938	-0.130	-0.033	0.019	0.146	0.034	-0.338
		(-4.61)	(4.61)	(-4.61)	(-0.95)	(-0.95)	(1.17)	(3.86)	(3.83)	(-2.79)
Standard Regular Mail	Non-Automated	0.327	-0.120	0.361	-0.255	-0.064	0.011	0.055	0.010	0.271
		(0.91)	(-0.91)	(0.91)	(-1.74)	(-1.74)	(3.44)	(3.44)	(3.44)	(3.44)
	Automated	0.078	-0.028	0.086	-0.061	-0.015	0.003	0.013	0.002	0.064
		(0.92)	(-0.92)	(0.92)	(-1.79)	(-1.79)	(3.85)	(3.85)	(3.85)	(3.85)
	Car-Rte Basic	0.204	-0.075	0.225	-0.160	-0.040	0.007	0.034	0.006	0.169
		(0.89)	(-0.89)	(0.89)	(-1.65)	(-1.65)	(2.96)	(2.96)	(2.96)	(2.96)
	Car-Rte HD&Sat	-0.024	0.009	-0.027	0.019	0.005	-0.001	-0.004	-0.001	-0.020
		(-0.36)	(0.36)	(-0.36)	(0.48)	(0.48)	(-0.55)	(-0.55)	(-0.55)	(-0.55)
Standard Nonprofit Mail	Non-Automated	-1.282	0.469	-1.414	-0.024	-0.006	0.014	0.068	0.012	0.334
		(-2.44)	(2.44)	(-2.44)	(-0.08)	(-0.08)	(1.96)	(1.96)	(1.96)	(1.96)
	Automated	-0.321	0.117	-0.354	-0.006	-0.002	0.003	0.017	0.003	0.084
		(-2.54)	(2.54)	(-2.54)	(-0.08)	(-0.08)	(2.02)	(2.02)	(2.02)	(2.02)
	Car-Rte Basic	-1.203	0.440	-1.327	-0.023	-0.006	0.013	0.064	0.012	0.314
		(-2.34)	(2.34)	(-2.34)	(-0.07)	(-0.07)	(1.91)	(1.91)	(1.91)	(1.91)
	Car-Rte HD&Sat	0.147	-0.054	0.162	0.003	0.001	-0.002	-0.008	-0.001	-0.038
		(0.70)	(-0.70)	(0.70)	(0.05)	(0.05)	(-0.67)	(-0.67)	(-0.67)	(-0.67)
Package Services	Parcel Post	0.258	-0.094	0.284	-0.225	-0.057	0.002	0.012	0.002	0.060
		(1.73)	(-1.73)	(1.73)	(-3.14)	(-3.14)	(1.83)	(1.83)	(1.83)	(1.83)
	Bound Printed Matter	0.052	-0.019	0.057	-0.045	-0.011	0.000	0.002	0.000	0.012
		(0.64)	(-0.64)	(0.64)	(-0.74)	(-0.74)	(0.65)	(0.65)	(0.65)	(0.65)
	Media & Library	0.091	-0.033	0.100	-0.079	-0.020	0.001	0.004	0.001	0.021
		(0.84)	(-0.84)	(0.84)	(-1.00)	(-1.00)	(0.86)	(0.86)	(0.86)	(0.86)

Elasticity of demand for product listed down the left with respect to the revenue per piece of the product listed across the top.
t-values in brackets (.)

Class	Category	First-Class Mail			Priority and Express		Periodicals			
		S-P	Non-Auto	Auto	Priority	Express	In-County	Nonprofit	Classroom	Regular
First-Class Mail	Single-Piece	-0.153	-0.396	-0.206	0.092	0.023	-0.002	-0.012	-0.002	-0.059
		(-0.86)	(-2.63)	(-1.17)	(3.16)	(3.16)	(-4.16)	(-4.16)	(-4.16)	(-4.16)
	Non-Automated	-12.243	-15.843	23.870	0.511	0.128	-0.013	-0.067	-0.012	-0.329
		(-2.55)	(-2.42)	(3.98)	(1.40)	(1.40)	(-1.49)	(-1.49)	(-1.49)	(-1.49)
	Automated	-0.199	0.774	-1.316	0.090	0.023	-0.002	-0.012	-0.002	-0.058
		(-1.05)	(3.95)	(-5.04)	(3.10)	(3.10)	(-4.03)	(-4.03)	(-4.03)	(-4.03)
Priority and Express Mai	Priority	0.312	-0.114	0.345	-1.120	0.019	-0.001	-0.005	-0.001	-0.027
		(2.42)	(-2.42)	(2.42)	(-12.60)	(0.55)	(-0.84)	(-0.84)	(-0.84)	(-0.84)
	Express	0.206	-0.075	0.228	0.573	-1.301	-0.001	-0.004	-0.001	-0.018
		(1.90)	(-1.90)	(1.90)	(2.02)	(-5.09)	(-0.78)	(-0.78)	(-0.78)	(-0.78)
Periodicals	Within-County	-0.907	0.332	-1.000	-0.138	-0.035	-0.102	-0.386	-0.163	0.503
		(-3.09)	(3.09)	(-3.09)	(-0.91)	(-0.91)	(-0.33)	(-1.60)	(-1.16)	(1.21)
	Nonprofit	-0.868	0.317	-0.957	-0.132	-0.033	-0.096	-0.791	0.029	0.716
		(-3.86)	(3.86)	(-3.86)	(-0.93)	(-0.93)	(-1.77)	(-5.81)	(0.96)	(3.30)
	Classroom	-0.423	0.155	-0.466	-0.064	-0.016	-0.569	0.212	-1.021	1.309
		(-1.00)	(1.00)	(-1.00)	(-0.57)	(-0.57)	(-1.29)	(0.50)	(-2.77)	(1.99)
	Regular Rate	-0.850	0.311	-0.938	-0.130	-0.033	0.019	0.146	0.034	-0.338
		(-4.61)	(4.61)	(-4.61)	(-0.95)	(-0.95)	(1.17)	(3.86)	(3.83)	(-2.79)
Standard Regular Mail	Non-Automated	0.327	-0.120	0.361	-0.255	-0.064	0.011	0.055	0.010	0.271
		(0.91)	(-0.91)	(0.91)	(-1.74)	(-1.74)	(3.44)	(3.44)	(3.44)	(3.44)
	Automated	0.078	-0.028	0.086	-0.061	-0.015	0.003	0.013	0.002	0.064
		(0.92)	(-0.92)	(0.92)	(-1.79)	(-1.79)	(3.85)	(3.85)	(3.85)	(3.85)
	Car-Rte Basic	0.204	-0.075	0.225	-0.160	-0.040	0.007	0.034	0.006	0.169
		(0.89)	(-0.89)	(0.89)	(-1.65)	(-1.65)	(2.96)	(2.96)	(2.96)	(2.96)
	Car-Rte HD&Sat	-0.024	0.009	-0.027	0.019	0.005	-0.001	-0.004	-0.001	-0.020
		(-0.36)	(0.36)	(-0.36)	(0.48)	(0.48)	(-0.55)	(-0.55)	(-0.55)	(-0.55)
Standard Nonprofit Mail	Non-Automated	-1.282	0.469	-1.414	-0.024	-0.006	0.014	0.068	0.012	0.334
		(-2.44)	(2.44)	(-2.44)	(-0.08)	(-0.08)	(1.96)	(1.96)	(1.96)	(1.96)
	Automated	-0.321	0.117	-0.354	-0.006	-0.002	0.003	0.017	0.003	0.084
		(-2.54)	(2.54)	(-2.54)	(-0.08)	(-0.08)	(2.02)	(2.02)	(2.02)	(2.02)
	Car-Rte Basic	-1.203	0.440	-1.327	-0.023	-0.006	0.013	0.064	0.012	0.314
		(-2.34)	(2.34)	(-2.34)	(-0.07)	(-0.07)	(1.91)	(1.91)	(1.91)	(1.91)
	Car-Rte HD&Sat	0.147	-0.054	0.162	0.003	0.001	-0.002	-0.008	-0.001	-0.038
		(0.70)	(-0.70)	(0.70)	(0.05)	(0.05)	(-0.67)	(-0.67)	(-0.67)	(-0.67)
Package Services	Parcel Post	0.258	-0.094	0.284	-0.225	-0.057	0.002	0.012	0.002	0.060
		(1.73)	(-1.73)	(1.73)	(-3.14)	(-3.14)	(1.83)	(1.83)	(1.83)	(1.83)
	Bound Printed Matter	0.052	-0.019	0.057	-0.045	-0.011	0.000	0.002	0.000	0.012
		(0.64)	(-0.64)	(0.64)	(-0.74)	(-0.74)	(0.65)	(0.65)	(0.65)	(0.65)
	Media & Library	0.091	-0.033	0.100	-0.079	-0.020	0.001	0.004	0.001	0.021
		(0.84)	(-0.84)	(0.84)	(-1.00)	(-1.00)	(0.86)	(0.86)	(0.86)	(0.86)

Elasticity of demand for product listed down the left with respect to the revenue per piece of the product listed across the top.
t-values in brackets (.)

Standard Regular Mail				Standard Nonprofit Mail			Package Services				Proportional
Non-Auto	Auto	CR Basic	CR HD&S	Non-Auto	Auto	CR Basic	CR HD&S	Parcel Post	BPM	Med & Lib	Row Sum
-0.048	0.082	-0.048	0.089	0.022	-0.058	0.023	-0.022	0.023	0.011	0.007	-0.636
(-0.96)	(0.96)	(-0.96)	(0.96)	(2.70)	(-2.70)	(2.70)	(-2.70)	(2.10)	(2.10)	(2.10)	(-1.93)
-0.268	0.460	-0.270	0.494	0.123	-0.323	0.131	-0.125	0.130	0.060	0.037	-3.550
(-0.74)	(0.74)	(-0.74)	(0.74)	(1.34)	(-1.34)	(1.34)	(-1.34)	(1.21)	(1.21)	(1.21)	(-0.06)
-0.047	0.081	-0.047	0.087	0.022	-0.057	0.023	-0.022	0.023	0.011	0.006	-0.625
(-0.96)	(0.96)	(-0.96)	(0.96)	(2.66)	(-2.66)	(2.66)	(-2.66)	(2.08)	(2.08)	(2.08)	(-1.54)
0.113	-0.194	0.114	-0.209	0.000	-0.000	0.000	-0.000	-0.067	-0.031	-0.019	-0.886
(1.35)	(-1.35)	(1.35)	(-1.35)	(0.01)	(-0.01)	(0.01)	(-0.01)	(-2.98)	(-2.98)	(-2.98)	(-2.65)
0.075	-0.128	0.075	-0.138	0.000	-0.000	0.000	-0.000	-0.044	-0.020	-0.013	-0.585
(1.21)	(-1.21)	(1.21)	(-1.21)	(0.01)	(-0.01)	(0.01)	(-0.01)	(-2.17)	(-2.17)	(-2.17)	(-1.28)
-0.620	1.062	-0.623	1.142	-0.088	0.232	-0.094	0.090	0.085	0.039	0.024	-0.647
(-2.91)	(2.91)	(-2.91)	(2.91)	(-1.84)	(1.84)	(-1.84)	(1.84)	(1.69)	(1.69)	(1.69)	(-0.65)
-0.593	1.017	-0.596	1.093	-0.085	0.222	-0.090	0.086	0.082	0.038	0.023	-0.619
(-3.53)	(3.53)	(-3.53)	(3.53)	(-2.00)	(2.00)	(-2.00)	(2.00)	(1.81)	(1.81)	(1.81)	(-0.91)
-0.289	0.495	-0.291	0.533	-0.041	0.108	-0.044	0.042	0.040	0.018	0.011	-0.302
(-0.99)	(0.99)	(-0.99)	(0.99)	(-0.86)	(0.86)	(-0.86)	(0.86)	(0.83)	(0.83)	(0.83)	(-0.21)
-0.581	0.996	-0.584	1.071	-0.083	0.217	-0.088	0.084	0.080	0.037	0.023	-0.606
(-4.08)	(4.08)	(-4.08)	(4.08)	(-2.10)	(2.10)	(-2.10)	(2.10)	(1.89)	(1.89)	(1.89)	(-1.10)
-1.160	-1.486	-0.820	0.224	-0.044	0.115	-0.046	0.045	0.091	0.042	0.026	-2.417
(-1.78)	(-1.49)	(-1.70)	(0.28)	(-0.91)	(0.91)	(-0.91)	(0.91)	(1.30)	(1.30)	(1.30)	(-1.49)
-0.050	-0.855	-0.043	0.178	-0.010	0.027	-0.011	0.011	0.022	0.010	0.006	-0.575
(-0.53)	(-3.99)	(-0.43)	(1.09)	(-0.93)	(0.93)	(-0.93)	(0.93)	(1.33)	(1.33)	(1.33)	(-1.72)
-0.312	-0.671	-1.764	0.724	-0.027	0.072	-0.029	0.028	0.057	0.026	0.016	-1.509
(-0.84)	(-1.02)	(-4.44)	(1.40)	(-0.89)	(0.89)	(-0.89)	(0.89)	(1.26)	(1.26)	(1.26)	(-1.41)
0.215	1.032	0.790	-1.797	0.003	-0.009	0.003	-0.003	-0.007	-0.003	-0.002	0.179
(0.70)	(1.49)	(2.66)	(-4.16)	(0.36)	(-0.36)	(0.36)	(-0.36)	(-0.43)	(-0.43)	(-0.43)	(0.19)
-0.372	0.638	-0.374	0.686	-2.580	2.337	-1.334	0.662	0.161	0.074	0.045	-1.886
(-0.88)	(0.88)	(-0.88)	(0.88)	(-6.62)	(2.99)	(-4.03)	(2.14)	(1.20)	(1.20)	(1.20)	(-1.04)
-0.093	0.160	-0.094	0.172	-0.243	0.031	-0.399	0.381	0.040	0.019	0.011	-0.472
(-0.89)	(0.89)	(-0.89)	(0.89)	(-3.92)	(0.23)	(-6.67)	(7.73)	(1.21)	(1.21)	(1.21)	(-1.15)
-0.349	0.599	-0.351	0.644	-1.167	1.138	-1.957	1.128	0.151	0.070	0.043	-1.770
(-0.87)	(0.87)	(-0.87)	(0.87)	(-2.81)	(1.19)	(-4.93)	(3.57)	(1.18)	(1.18)	(1.18)	(-0.95)
0.043	-0.073	0.043	-0.079	-0.149	0.902	0.374	-1.021	-0.018	-0.008	-0.005	0.216
(0.45)	(-0.45)	(0.45)	(-0.45)	(-0.42)	(1.07)	(1.22)	(-3.83)	(-0.54)	(-0.54)	(-0.54)	(0.20)
-0.151	0.258	-0.152	0.278	-0.030	0.079	-0.032	0.031	-1.165	-0.155	-0.118	-0.912
(-1.29)	(1.29)	(-1.29)	(1.29)	(-1.25)	(1.25)	(-1.25)	(1.25)	(-19.47)	(-2.90)	(-2.97)	(-2.11)
-0.030	0.052	-0.030	0.056	-0.006	0.016	-0.006	0.006	0.168	-0.246	-0.211	-0.183
(-0.57)	(0.57)	(-0.57)	(0.57)	(-0.56)	(0.56)	(-0.56)	(0.56)	(1.20)	(-0.66)	(-0.91)	(-0.36)
-0.053	0.091	-0.053	0.098	-0.011	0.028	-0.011	0.011	-0.136	-0.345	-0.026	-0.321
(-0.72)	(0.72)	(-0.72)	(0.72)	(-0.71)	(0.71)	(-0.71)	(0.71)	(-0.67)	(-0.89)	(-0.07)	(-0.51)

Elasticity of demand for product listed down the left with respect to the revenue per piece of the product listed across the top.
t-values in brackets (.)