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BEFORE THE
POSTAL REGULATORY COMMISSION
WASHINGTON, DC 20268-001

Notice of Market-Dominant Price Adjustment	)	Docket No. R2013-1

Comments of the

American Catalog Mailers Association (ACMA)

(November 1, 2012)

On October 11, 2012, the Postal Service noticed plans to adjust the prices of its market-dominant products. In Order No. 1501 (October 15, 2012), the Commission invited comments. ACMA is pleased to sponsor these comments, accompanied by one Excel workbook, ACMA_R2013-1_Workbook.xlsx.

Through catalogs, ACMA's members make a wide range of goods and services available to businesses and consumers. Catalogs are valued highly by mail recipients and bring content to the mailstream that helps consumers and businesses engaged with mail. Catalogs are read and used by the vast majority of those who receive them. *See* Appendix I, a summary of findings from an independently-conducted national research study commissioned by ACMA.

Depending on the proportions of ZIP Codes and carrier routes covered, they use rates in Standard Flats, Carrier Route flats, and High-Density flats. Typically, postage represents 40 to 60 percent of their marketing costs. On the postal side, catalogs account for a high proportion of the flats in the Standard class. Therefore, the rates at issue are critically important to both catalogers and the Postal Service.

I. ACMA's Position

ACMA provides no comments on the specific rates proposed by the Postal Service. However, we continue to be concerned, and to share the Postal Service's concern,[1] that the costs being reported for flats, including Periodicals, lack the robustness that should be required, and, accordingly, that perspective is unduly limited on the relation of the rates to the costs.[2]

We have provided evidence on associated costing issues in past proceedings. We supplement, update, and summarize that evidence here.

II. Costing Guidance

When decisions on rates are made, it is fundamental that information be available on the *effects* of selecting one rate instead of another. One of the effects may

[1] *See* Notice of Market-Dominant Price Adjustment, October 11, 2012 at 22, where the Postal Service points to "the inflexibility of union contracts" and to costs being uncorrelated with volume. Despite these concerns, however, the Postal Service, conservatively, treats the reported costs as variable with volume in its contribution model (Standard Mail Contribution Model (USPS-LR-R2013-1(7).xls).

[2] On October 22, 2012, Valpak filed a "Motion to Strike [the] Standard Mail Price Adjustment from" the Postal Service's filing. Strangely, Valpak does not address the validity of the costs underlying its motion, despite the fact that questions on the costs have been raised by ACMA on numerous occasions. It would seem, however, that the robustness of the costs should be a consideration in evaluating the extent to which any category of flats is an "extreme case" of the kind discussed by the court in USCA Case #11-1117, opinion April 17, 2012.

be a change in costs.[3] That is, for example, a lower rate instead of a higher rate might be expected to result in higher costs, if the lower rate causes the volume to increase. Since such volume changes (often referred to as *rate-induced* volume changes) are rather small, attention centers on small-volume-change incremental costs, on a unit basis, often approximated by estimates of marginal costs. Importantly, such a focus is consistent with prescriptions derived from notions of economic efficiency.

Costing in economics presumes the existence of "a *causal relationship* between the quantities of [the] various economic good and services provided and the expenditures incurred by the entity producing those goods and services."[4] Such relationships can exist for costs that are at the efficient level (however "the efficient level" is defined) or for costs that are above the efficient level. "[I]t is only necessary that there exist[ ] a relatively stable relationship between inputs, outputs, and factor prices …." *Id*. at 3. If a stable relationship does not exist, meaningful deterministic models cannot be built and meaningful costs cannot be estimated.

Difficulties can be of three types. First, a stable relationship could exist but be poorly modeled. Statistical tests of fit are usually used to help in modeling efforts. Second, a relationship could change between the time the model is built and the time the model is used, caused, possibly, by changes in volume, changes in technology, or changes in the network.[5] Third, a relationship may not exist, such as might be caused

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[3] Other effects exist, of course, including "the effect[s] … upon the general public, business mail users, and enterprises in the private sector of the economy engaged in the delivery of mail matter other than letters" (39 U.S.C.§ 3622(c)(3)).

[4] See John C. Panzar, "Costs for Better Management Decisions: CRA Versus Fully Distributed Costs," Postal Service OIG Report No. RARC-WP-12-016, September 17, 2012, text at 3.

[5] The existence of volume declines is evident. The Postal Service mentions Network Rationalization as an operational change. A volume reduction could occur within a given scale of operations. Network changes could change the scale. Having 10 plants instead of 16 plants might not be

by the presence of excess capacity. Excess capacity in the Postal Service has been acknowledged and discussed recently.[6]

III. Assessing Results

At the time models are built and costing procedures are developed, attention centers on assuring their validity. The statistical tests go only so far, however, and often leave much to be desired. But after applying the procedures for a period of time, a powerful dimension is added to the toolkit available. Specifically, perspective on what is happening in operations[7] and on how costs *should behave* often provide considerable insight into the validity of the procedures and therefore of any estimates derived

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a change in scale. That is, each plant is the same scale as before. A change in scale would occur when the plants are knit together into a network. Also, the management pyramid involved could be smaller.

[6] *See* "A Primer on Postal Costing Issues," USPS Office of the Inspector General, Report RARC-WP-12-008, March 20, 2012, at i, which asks: "How should the [costing] system be adapted to reflect the excess capacity currently present in the postal network …?"

In the presence of excess capacity, the relationship that exists between volume and costs is, in effect, a nil relationship. That is, an increase in volume would cause a zero increase in costs and a decrease in volume would allow no further decrease in costs. A prudent manager would consider such a situation in decision-making, regardless of how accountants might be allocating costs. It is possible, of course, for excess capacity to exist in some processing and delivery areas and not others.

[7] For example, in reply comments relating to parcel costs, the Postal Service stated: "There are no operational explanations as to why Carrier Route parcels would have such high delivery costs." Reply Comments, August 10, 2010, Docket No. RM2012-5. A similar comment could be that no operational reasons explain why a cost increase should be so large.

therefrom.[8] It may be difficult and expensive to develop new procedures, but that is another matter.[9]

IV. What ACMA Has Found

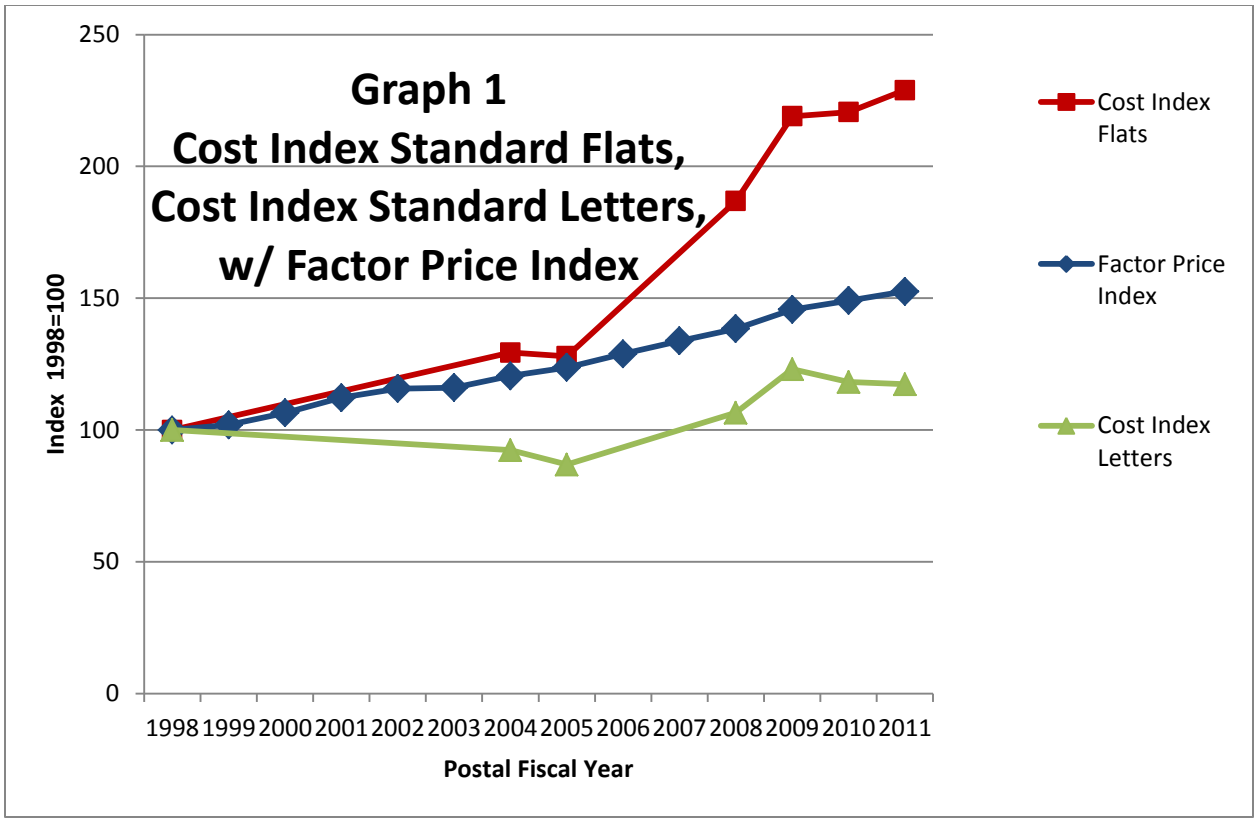
For use at aggregate levels, such as the level of a product, ACMA has developed a cost index, explained in detail in its initial comments in Docket No. ACR2011. Basically, it is an index of unit attributable costs corrected for the effects of changes in the relative proportions of the encircled volume elements, i.e., of the mix of the volume.[10]

ACMA developed cost indexes for Standard Flats (since 1998), Carrier Route (since 2008), Periodicals (since 1997), and, to allow comparison, Standard Letters (since 1998). The findings were consistent across the flats categories. Graph 1 shows the cost indexes developed for Standard Flats and Standard Letters, along with the factor price index (the latter updated to 2011), for the 1998-2011 period. Factor prices,

[8] Suppose it is understood that there have been no meaningful changes in volume characteristics or operations, and factor prices increase 4%. The expectation would be that costs increase by approximately 4%. If the cost reported shows an increase of 4.5%, it may be decided that there is no basis for questions. But if costs increase 7.5%, there would be a basis for raising questions about the costs, perhaps suggesting excess capacity. The proof is in the pudding, not in statistical tests done over an observation period used to build the model.

[9] Note that developing new procedures is not an end in itself. As they are developed, the same questions relating to their validity are faced that were faced when the procedures already in place were developed. And even if the new ones pass muster, a new dimension will be added when time passes.

[10] Consider an example. From period 1 to period 2, suppose the piece volume remained at 1,000 pieces. But suppose the mix in period 1 was 40% 5-digit and 60% no-sort, and in period 2 it was 70% 5-digit and 30% no-sort ("no-sort" meaning that outgoing, incoming, and incoming secondary sorts are required, at least, and 5-digit meaning that an incoming secondary only is needed). The increase in presortation should cause the unit cost to decline, say, 25%. If the reported unit cost increased 5%, one would say that costs increased 30%, approximately. Index number procedures were used by ACMA to quantify this increase in costs.



which transmit pressure to all categories of costs, increased 52.6 percent. However, the cost of Letter increased only 17.3 percent and the cost of Flats increased a whopping 128.9 percent. An analysis of the reasons for these outcomes has not been performed, but is warranted.

From 1998 through 2005, factor prices increased 23.7 percent, but the cost of Letters *decreased* 13.2 percent. This means that Letter costs were 29.8 percent lower than they would have been if they had tracked factor prices. During this period, the DPS rate went from about 61 percent to about 83 percent.[11] This is a notable increase, causing a cost reduction on approximately 22 percent of the volume (83% - 61%), but it

[11] DPS percentages and sources are shown on the "Std Letters" tab of the workbook ACMA_R2013-1_Workbook.xlsx.

falls short of explaining a decrease in costs of 29.8 percent.[12] Something else must be happening, and it must be sizable. From 2005 through 2011, the cost of Letters increased along with the factor prices, despite a reduction in volume of about 9.4 percent.[13] This is in line with what one might expect.

The results for Flats are considerably different. From 1998 through 2005, the cost of Flats increased with factor prices, despite improvements in automation and mail preparation. Then from 2005 through 2011, a period of just 6 years, their cost increased 78.3 percent while the factor prices increased only 23.3 percent. At the same time, the volume of flats decreased 28.3 percent.

It is true that the reduction in volume was greater for flats than for letters, but it is not clear that a reduction in volume should have a substantial effect. First, mail processing operations are treated as 100 percent variable, or nearly so, so the unit costs of these should not be affected by volume, certainly not under longer-run costing, which requires full adjustment to volume changes. If the rise in costs is taken to mean that the costs are not 100 percent variable, then the costs are erroneous. Second, most of the fixity is in carrier operations, but the regressions underlying carrier costing have not been updated and the scale of carrier operations is relatively unchanged. We believe the results are consistent with costs of excess capacity being attributed to flat-shaped mail, which means the costs are not relevant for rate purposes.

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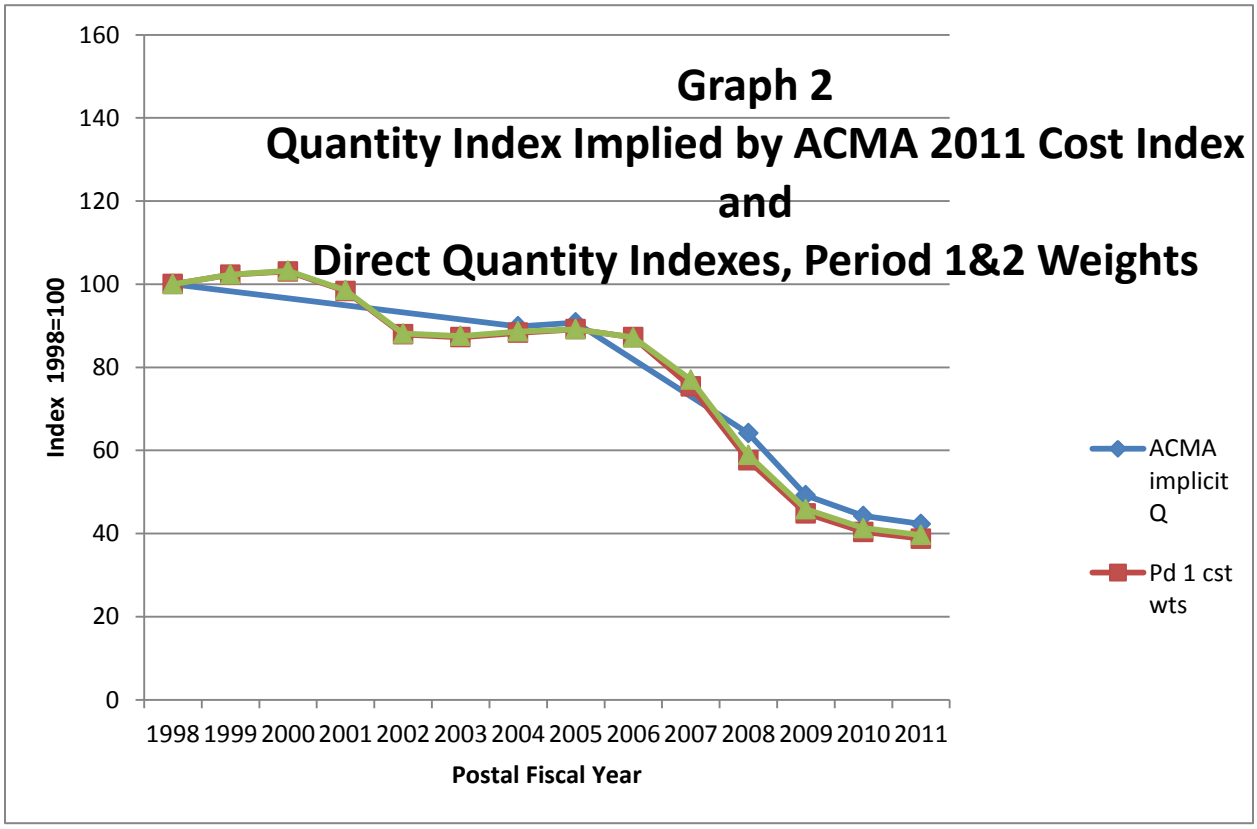
[12] So far as we know, the record does not contain an estimate of the cost reduction that is allowed by DPSing. A reduction should occur in carrier casing time and an increase should occur in mail processing cost. The net savings, occurring on approximately 22 percent of the volume, would not likely account for a decrease in the overall cost for letters of 29.8 percent.

[13] Volume changes are developed on tab “Volumes” of the workbook ACMA_R2013-1_Workbook.xlsx.

V. Comments of the Public Representative on ACMA's Cost Index

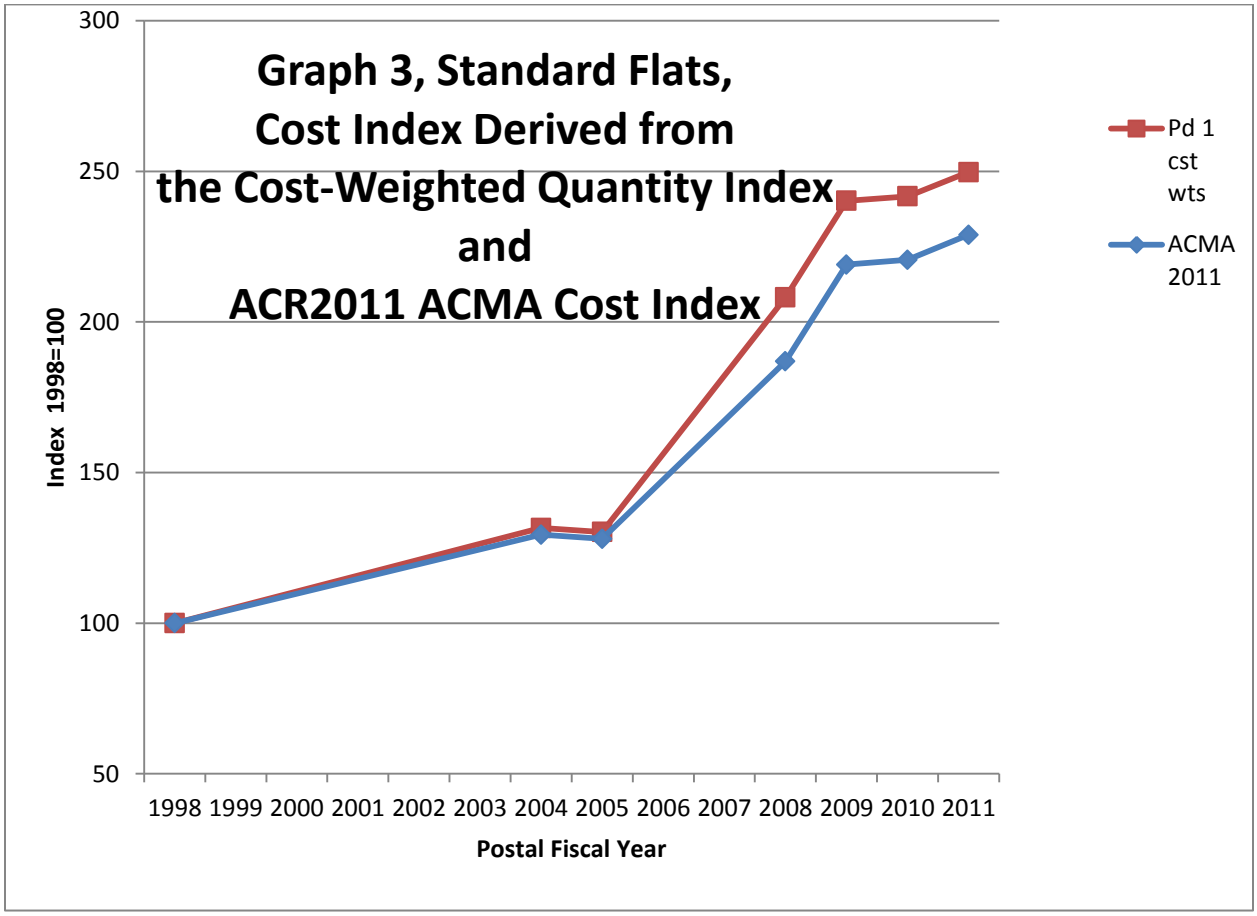
In reply comments in Docket No. ACR2011, the Public Representative made a number of observations relating to ACMA's cost index. Appendix II to the instant comments explains that the Representative's observations are not useful. Generally, they are underdeveloped, misguided, or wrong. However, the Representative touched opaquely on an issue ACMA mentioned in its ARC2011 initial comments, that a cost index derived from a *cost*-weighted quantity index might be more indicative than one reflecting a *price*-weighted quantity index (ACMA's cost index being of the latter kind).

To explore the question of the effect of using a *cost*-weighted quantity index, we developed one for the period covered by the earlier analysis. Doing so is not a trivial exercise. *See* tab "Std Quant" in ACMA_R2013-1_Workbook.xlsx, which aligns volume categories with costs available. The results are displayed in Graph 2, which shows that



since FY 1998, the price-weighted quantity index implicit[14] t in the ACMA cost index for Standard Flats (labeled "ACMA implicit Q") decreased 57.7 percent while the corresponding *cost*-weighted quantity index decreased 60.4 percent using period-2 weights (labeled "Pd 2 cst wts") and 61.2 percent using period-1 weights (labeled "Pd 1 cst wts"). A cost-weighted *cost index* is obtained by dividing these indexes into an index of total costs.

Graph 3 shows the cost index derived from the cost-weighted quantity index (using period-1 cost weights, labeled "Pd 1 cst wts"), along with the ACMA cost index of ACR2011 (labeled "ACMA 2011"). Since the cost-weighted quantity index is lower than



[14] As shown in the Appendix of ACMA's ACR2011 initial comments, the cost index is equivalent to a simple index of total costs divided by a price-weighted quantity index. Implicitly, then, that price-weighted quantity index can be found by dividing the index of total costs by the cost index.

the implicit price-weighted quantity index, the associated cost index is higher, suggesting that ACMA's cost index may have been low for Standard Flats by 15 to 20 percentage points. That is, over the period 1998 – 2011, the cost increase for Standard Flats may have been 15 to 20 percentage points higher than shown by the ACMA index. Thus, the situation surrounding costs might be notably worse than ACMA's cost index showed.

VI. Supporting, Less-Aggregate Findings

Examining components underlying aggregate findings can improve the understanding of them. ACMA has examined a number of components and finds them to support the results shown by the cost index.

1. In Docket No. R2006-1, based on costs reported for (FY) 2005, a detailed, systematic projection for 2008 was made of the mail processing and delivery costs of Mixed ADC letters and flats in Standard Mail.[15] For 2008, here are the projections and the costs that were later reported, along with the cost reported for flats in 2009.

	Costs in cents/pc for Mixed ADC Letters & Flats				
	08 projected	08 actual	% higher	09 actual	y/y % inc
Letter	10.0 ¢	10.5 ¢	**5.0%**		
Flat	38.0 ¢	51.6 ¢	**35.8%**	59.2 ¢	**14.7%**
ratio	**3.8**	**4.9**			

A flat costs 3.8 times as much as a letter in the projection (38.0¢/10.0¢), but the actual outcome was 4.9 times as much (51.6¢/10.5¢). Stated another way, the cost of letters increased 5 percent (10.5¢/10.0¢) and of flats increased 35.8 percent (51.6¢/38.0¢).

[15] These projections, done in what was called a "roll forward" process, recognized forecasts of inflation and other factor prices (generally relying on projections made by an outside, independent, forecasting service), volume, union agreements, and Postal Service programs to change equipment and technology.

Then in FY 2009, flats increased another 14.7 percent (59.2¢/51.6¢). These results raise serious questions about the costs being reported for flats. ACMA Initial Comments at 3-5, Docket No. R2010-4.

2. In our initial comments in Docket No. ACR2010, we showed that the city carrier costs for FY2010 imply that it takes about "3.8 seconds longer to case a Regular Flat than to case a Carrier Route flat" (at 11). This difference is greater than an understanding of carrier operations would suggest. Also, we showed that it is significantly higher than suggested by an earlier Postal Service study.

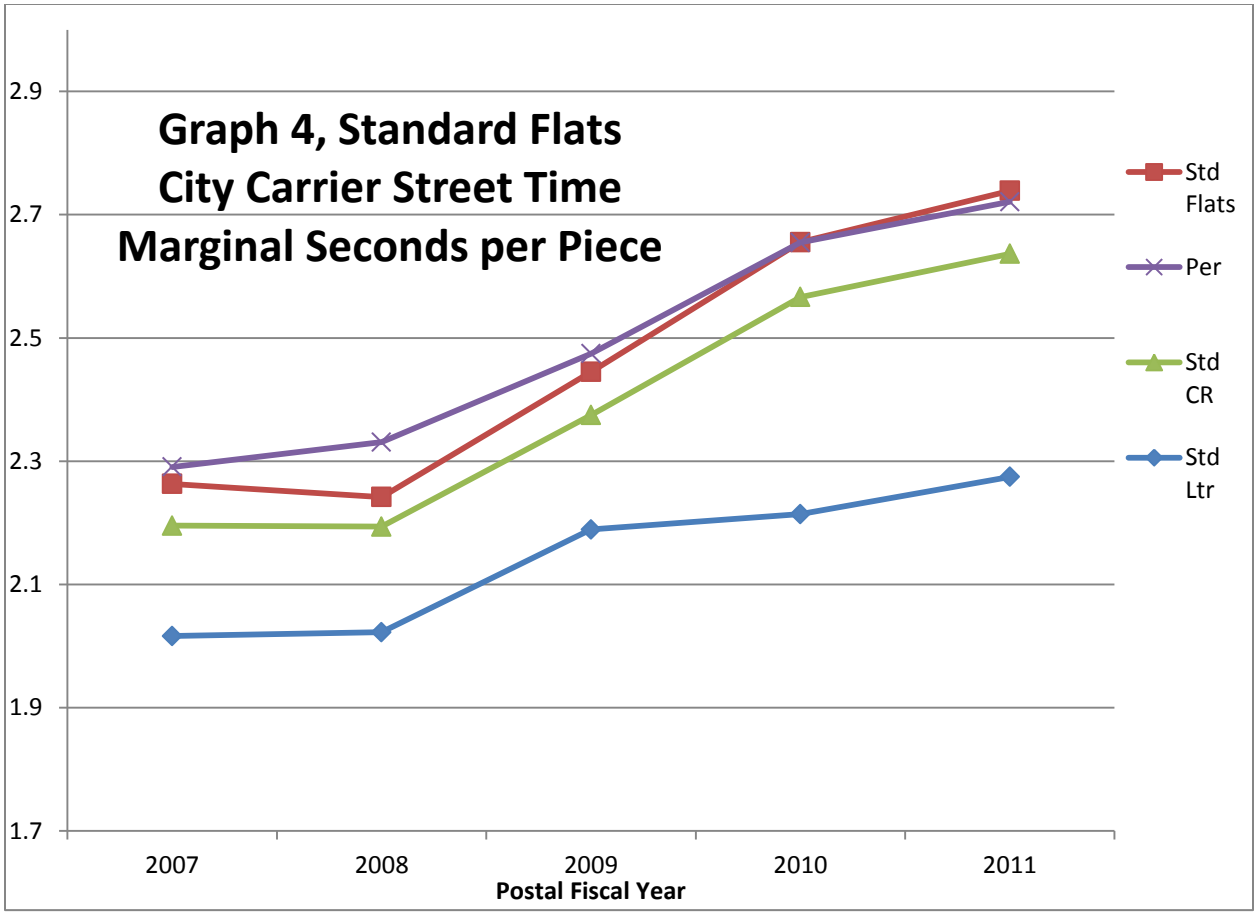
3. In our reply comments in Docket No. ACR2010 (at 5-7), we showed inexplicably large increases from FY 1998 to FY 2010 in unit mail processing and delivery costs for 5-digit automation flats, and provided a number of reasons why the increases we found might be an understatement.

4. In our comments in Docket No. R2011-2 (at 4-6), we showed an inordinately high ratio between the cost of casing Standard Letters and Saturation letters, and between the mail processing costs of 5-digit automation flats and Carrier Route flats. On the latter: the 5-digit automation flat receives one more sort than the Carrier Route flat, yet its cost is 15.5 cents higher. These differences are much too large and raise serious questions that have not been addressed.

5. In our statement in Docket No. R2012-3 (at 6-11), we reviewed the city carrier office and street costs for Standard Flats and Standard Letters for the years FY 2005 to FY 2010. The patterns and sizes of the increases were disturbing. The street costs are discussed further below.

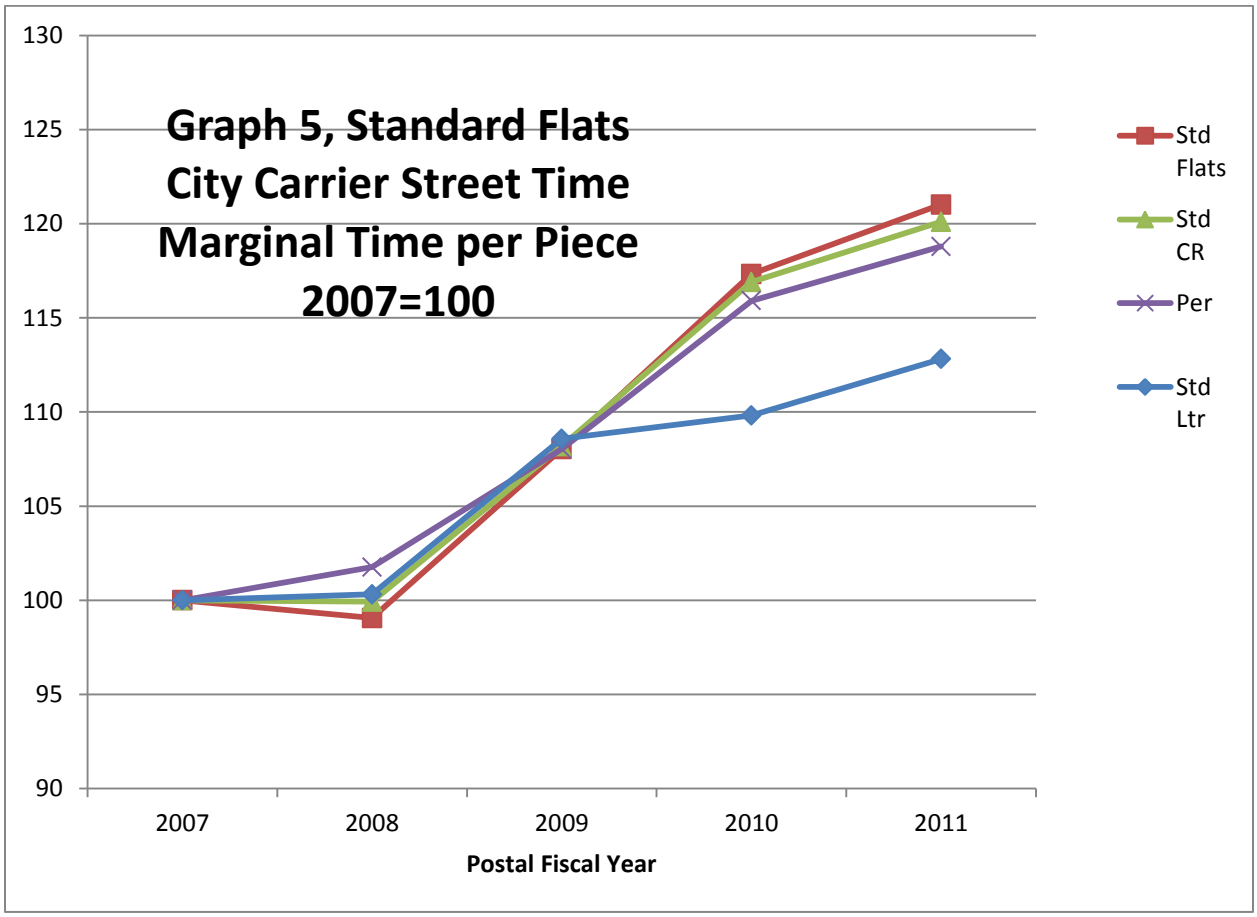
VII. City Carrier Street Costs

Cost segment 7.1 shows the direct street costs for city carriers. The costing is the result of a special data collection and associated regressions in Docket No. R2005-1. We have assembled data from FY 2007 to FY 2011, and have used carrier wage rates for each year to convert the unit costs into marginal seconds per piece. Though elemental and small in and of themselves, marginal seconds get multiplied by billions of pieces, and then magnified when piggyback costs and indirect costs are layered on. Therefore, they account for a significant proportion of the costs reported. Graph 4 shows these marginal times for Standard Flats, Carrier Route flats, Periodicals, and Standard Letters.



Several questions arise. The R2005-1 Recommended Decision (at 68) shows additional times of 1.7 seconds for both letters and flats, but in the FY 2007 CRA, letters had crept up to 2.0 seconds and flats to as high as 2.3 seconds. Then by FY 2011, the additional times ranged from 2.3 to 2.7 seconds. If one thinks of 1.7 seconds as being characteristic of the behavior of the delivery system (additional time to finger an additional piece, to carry an additional tray when needed, and so forth), these increases are startling. We know of no changes in operations that might explain them.

Graph 5 shows the marginal times, indexed to FY 2007.



It is clear that the flats times increased together. However, the letter time increased 12.8 percent and the flats times increased as much as 21.0 percent. If the

regressions had been updated, one could surmise that the marginal times might reflect additional access costs, due possibly to lower volume. But the regressions have not been updated. Another possibility could be that a constant percent variability has been assumed, but there is no empirical justification for such an assumption. Making such an assumption anyhow can lead to the distribution of non-volume-variable costs[16] or to the distribution of costs of excess capacity, or both, even though such costs would not be suited to rate purposes. Additional inquiry is needed. The footing available for drawing conclusions about rates is too much like sand.

VIII. Conclusion

Once costing systems are in place, assessments of their behavior over time can be a powerful check on their validity. Short of a laboratory experiment, nothing compares to letting nature play itself out. To help with such assessments, ACMA has developed a cost index, which quantifies the increases in reported costs, corrected for changes in volume mix. One check on this index, developed in section V above, suggests that it might be an understatement.

ACMA has applied its cost index to Standard Letters, Standard Flats, Carrier Route, and Periodicals. To the point of the costs being anomalous, the finding is that the increases in reported flats costs are substantially out of line with the increases implied by an understanding of operations and a knowledge of factor prices. This finding is bolstered by an examination of a number of component costs. At the very

[16] In a productive operation where the fixed costs are identifiable and separable, as in route time and, to a considerable extent, in travel time and access time, a volume decline would be expected to be associated with a *decrease* in the variability percent. Note that the development of longer routes, allowed by DPSing, would probably decrease the fixed cost of travel time, but a new variability study would be needed to capture this effect.

least, increases of the magnitude observed should be explained, and they have not been. Accordingly, the support provided for any conclusion about the relation of flats rates to flats costs is critically weak. In effect, the dimensions of that relation are not known.

Since the Commission's finding that Standard Flats are out of compliance with the statute because they do not cover their costs is rooted in a presumption that the cost data are accurate, ACMA feels that the numerous and important questions on cost validity raised in our submissions must be addressed before action is taken based on those costs. What is more, it is clear that catalogs add overall appeal to the mail from the consumer perspective. The loss of catalogs from the mail would harm the impact and value of mail to the recipient, reducing the strength of the mail channel as a marketing medium for all mailers. There, we respectfully request that the Commission or the Postal Service investigate the issues raised by ACMA and that, until these questions are answered, a conservative approach to forcing increases in Standard Flats rates be taken.

Respectfully submitted,

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Appendix I to ACMA Comments, Docket No. R2013-1, November 1, 2012

(three pages follow this one)

Catalogs: The Consumers' Point of View

A Survey Conducted by FGI Research

Commissioned by American Catalog Mailers Association (ACMA)

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Dino Fire, Director of Marketing Science, FGI Research
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Background

In a nationwide telephone survey among a random sample of 817 consumers ages 18+ to understand consumers' opinions and behaviors on catalogs, the value of catalogs to consumers was quantified including their value to those that use them frequently. The research fielded in and around the 2011 Holiday Shopping period. This white paper provides a summary of the key findings as well as the results of predictive modeling of the survey data that catalogers and other mailing interests may find helpful.

Higher prices for most postal mailing and shipping services went into effect on January 22, 2012.[i] Increases such as these have a direct negative impact on the catalog industry, making it more expensive to get catalogs into the hands of consumers. The Postal Service claims that 81 percent of American households surveyed in 2010 reported that they either read or scanned advertising mail.[ii]

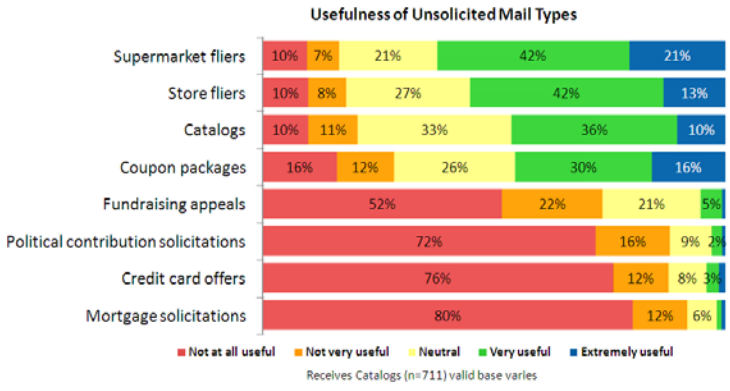
Key Findings

Consumers who receive and use catalogs consider them far more useful than many other types of unsolicited mail. Respondents who receive catalogs by mail, say they open and look at two-thirds of the three catalogs they receive (on average) per week. More than half do so as soon as they arrive in the mail. Virtually all consumers who receive catalogs have made a purchase from a company whose catalog they receive in the mail, with half doing so within the past month. The survey shows that consumers most commonly make these purchases by first reviewing their catalogs, then making purchases through the companies' websites. Respondents most often shop with companies whose catalogs they receive to get hard-to-find items not found in stores.

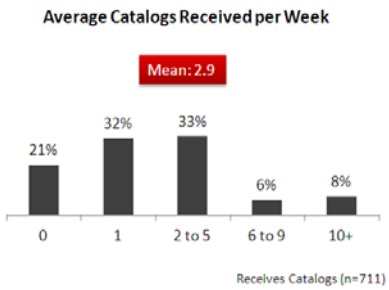
When it comes to unsolicited mail, catalogs are considered far more useful than many other types of mail including fundraising appeals, political contribution solicitations, credits card offers, and mortgage solicitations. Nearly half (46%) of those who receive catalogs find them useful, compared to only 5% or less for the aforementioned other specific types of mail. Other findings:

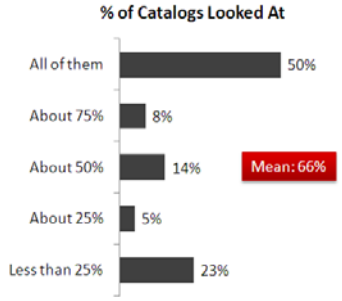
- ✓ Ages 55+ receive more catalogs on average than ages 18-54.
- ✓ HHI $100K + receive more on average, however HHI <$50K open up and look at a higher percentage of those received.
- ✓ Women open up and look at a higher percentage of catalogs received than men.
- ✓ Half of respondents most often shop from catalogs during the holiday period.
- ✓ When asked the reasons they shop catalogs, hard to find items was selected most frequently, followed by ease of shopping quality and greater assortment

Supporting Research



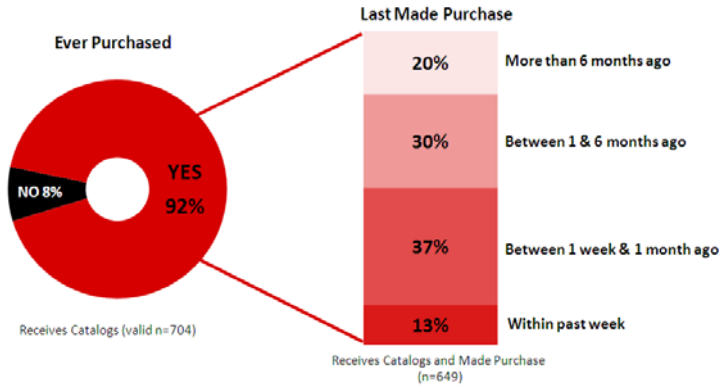
Catalogs are considered useful by 46% of those who receive them, comparable to supermarket and other retail flyers. This is vastly different than political contribution solicitations, credit card offers, and mortgage solicitations. These latter items are not found to be useful by nearly everyone who gets them.

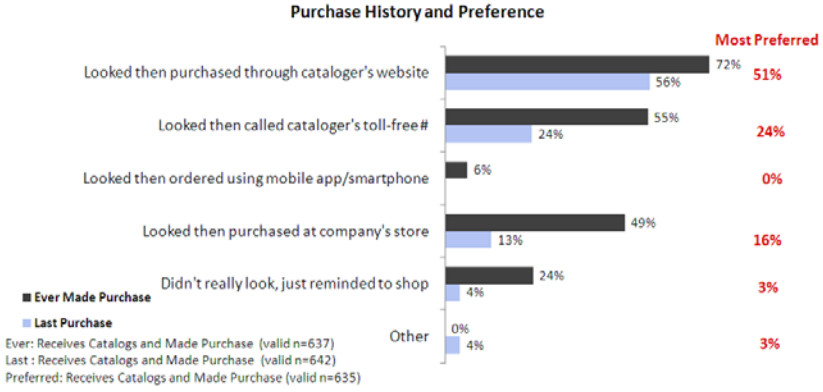




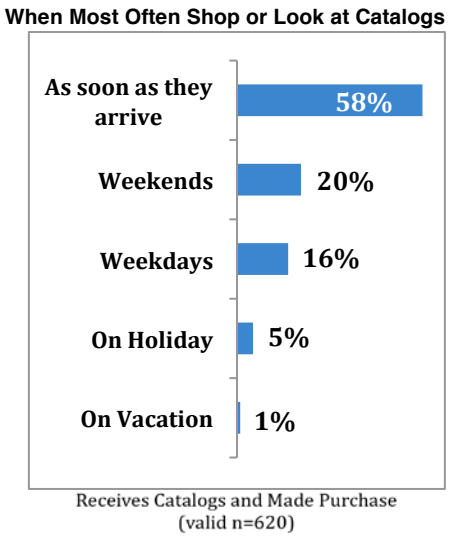
Among those who receive catalogs by mail, about three catalogs are received per week on average. Two-thirds of these catalogs received are opened up and looked at, with half of respondents opening up and looking at <u>ALL</u> of the catalogs they receive.

Nearly everyone who received catalogs has made a purchase from those catalogs. Half have done so within the past week or past month.

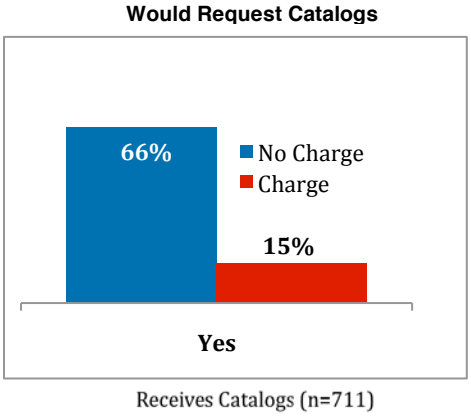




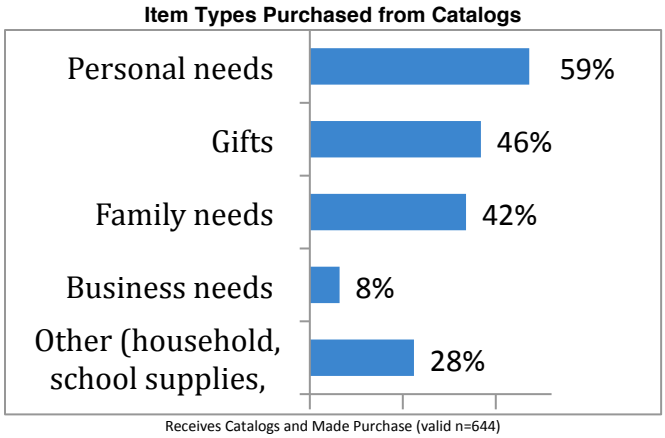
The <u>most used</u> and <u>preferred</u> method for purchases from a company whose catalog was received was to look at the catalog and then purchase through their website. This method is preferred twice as much as using the toll-free number.



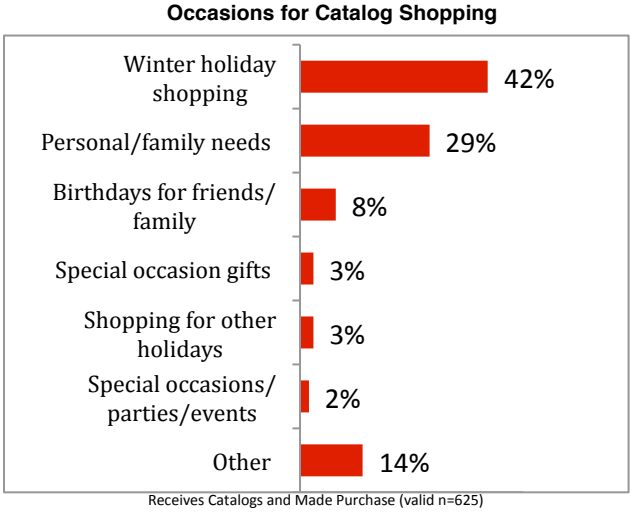
✓ *More than half of respondents look at catalogs as soon as they arrive.*



If catalogs became so expensive to produce and mail that they could only be received through a request to the merchant or through a service, two-thirds would request them. More than 1 in 7 consumers would actually pay to receive catalogs.



✓ *Older Americans are more likely to shop year round and for personal needs, as are men of varying ages.*
✓ *More women than men shop during the holidays. This is also true for ages 35-54, compared to ages 55+, and those who have children in the household compared to those without children.*



Predicting Consumer Behaviors

On average, consumers who receive catalogs spend $850 per year on catalog purchases. The median annual spend is $347 per year. This variation in statistics indicates that those who spend more than the median amount spend much more.

Conclusions

Catalogs certainly provide value to consumers; indeed, about 90% of consumers receive catalogs. Most find them valuable and read and use them. Catalogs provide an important shopping channel for certain types of consumers and during many times of the year.

About FGI Research

For 30 years, FGI has conducted custom research across a wide variety of industries. In addition to providing research solutions to major retailers and manufacturers, FGI's previous experience includes membership-based organizations such as AARP, NEA, NC Dental Society, the NC and American Bar Association, as well as the UNC University System and various departments within UNC-Chapel Hill. In addition, we have done work for Capstrat, a former sister company who lobbies on behalf of a number of major North Carolina clients. To learn more, please contact John Blunk, Director of Client Services at (919) 932-8847 or jblunk@fgiresearch.com. Learn much more about us at www.fgiresearch.com.

References

[i] http:://about.usps.com/news/national-releases/2011/pr11_factsht_pricechng_1018.pdf
[ii] http://www.nytimes.com/2011/12/04/sunday-review/the-junking-of-the-postal-service.html?pagewanted=2&_r=3&ref=opinion

Appendix II to ACMA Comments, Docket No. R2013-1, November 1, 2012

Response of Robert W. Mitchell to

ACR2011 Reply Comments of the Public Representative

Introduction

In the Appendix to its Initial Comments in Docket No. ACR 2011 (February 3, 2012), ACMA presented my proof that the ratio of a Laspeyres price index for a period 2 divided by the period-2 cost coverage to the corresponding Laspeyres price index for a period 1 divided by the period-1 cost coverage is a cost index. In his Reply Comments in the same docket (February 17, 2012), the Public Representative (hereinafter "Representative") addressed my price index and the resulting cost index.

Part I below responds to his comments on my price index, which is integral to my cost index. Part II responds to his comments on the cost index itself. On each matter, the Representative's thoughts are underdeveloped, misguided, or wrong. No case has been made that my cost index is defective.

Implicit in my cost index is a *price*-weighted quantity index. It is true that relying on a *cost*-weighted quantity index might be more indicative. For this reason, one is presented in the text of the comments to which this Appendix is appended. For the period analyzed, it suggests that my cost index might have been an understatement.

Here is my central equality, which I proved, and which the Representative did not question. I will refer to its left-hand side (LHS) and its right-hand side (RHS).

$$\frac{\frac{Laspeyres\ price\ index_2}{Cost\ coverage_2}}{\frac{Laspeyres\ price\ index_1}{Cost\ coverage_1}} = \frac{Total - cost\ ratio_{2/1}}{Price - weighted\ quantity\ index_{2/1}}$$

The LHS is my cost index. The RHS shows the interpretation of the LHS. In computations, I followed the LHS only. The "total-cost ratio" is a simple quotient—no weights needed. The periods may be of any length, need not be of the same length, and need not be adjacent, though I assume adjacency herein. Generally, P = price, uppercase for a product and lowercase for a price element. I use the term *price* instead of the term *rate*, reserving the latter to refer to growth rates.

PART I –THE REPRESENTATIVE'S COMMENTS ON MY PRICE INDEX

The Representative begins with an observation on my price index, as contained in the LHS of my central equality, saying:

> First, [ACMA's] [price] index … is measured by the percentage increase in [prices] from the previous year:
>
> $$RI_t = \left(\frac{P_t}{P_{t-1}} - 1\right)$$
>
> Where RI_t is the price index in period t, and (P_t/P_{t-1} - 1) is the annual percentage change in price. ACMA's measurement of the [price] index is flawed however. The proper formula for a continuous [price] index is:
>
> $$\ln\left(\frac{P_t}{P_{t-1}}\right)$$
>
> This formula avoids calculating different percentage changes depending on which year is the base year. However, the error is small.

P. 9-10, footnote omitted, inserting the term *price* for the term *rate* (4 places).

Before going further, a degree of clarification is needed. It seems apparent that the two separate-line formulas in this quotation were meant to be, respectively: $RI_t = RI_{t-1} * (\frac{P_t}{P_{t-1}})$ and $RI_{t-1} * (1 + \ln\left(\frac{P_t}{P_{t-1}}\right))$. If it were the base, RI_{t-1} would be set equal to

100. Alternatively, if taken as understood that $RI_{t-1} = 1.0$, the two formulas would be: $RI_t = \left(\frac{P_t}{P_{t-1}}\right)$ and $1 + \ln\left(\frac{P_t}{P_{t-1}}\right)$. Either way, my price index reflects the price relative $\frac{P_t}{P_{t-1}}$ (or an index-number estimate of it), but does not contain any percentage increases. Also, when the Representative refers to the "proper formula for a continuous [price] index," I assume he means the proper formula for the growth rate of a continuous price. Accordingly, I interpret the Representative as suggesting that logs be taken of price relatives or index-number estimates of them.

Thus clarified, a number of considerations suggest that the Representative's criticism is problematic—certainly it is not a matter of "proper" vs. improper, and his use of the term "flawed" is unwarranted. Also, saying "the error is small" does not help.

What is the Meaning of the Representative's Measure?

Suppose P_{t-1} is 20 and P_t is 25. The increase is 5. Relative to P_{t-1}, the increase is 25 percent ($100 * \frac{5}{20}$); relative to P_t, it is 20 percent ($100 * \frac{5}{25}$). Under the latter quantification, it would be said that the increase is 20 percent of the new level.[1]

The *logarithmus naturalis* (hereinafter log or ln) of $\frac{P_t}{P_{t-1}}$ is $\ln(25/20)$ = 0.223144 (rounded to 6 decimal places), which, on a per-100 basis, is 22.3144%. This is the measure suggested by the Representative. Its meaning is: *If, immediately after P_{t-1} is implemented, it begins to grow at a per-period rate of* $\ln(25/20)$*, compounded*

[1] Expressing the size of an increase as a proportion of the new level is uncommon, though not wrong. For example, a man receiving a 3.5% pay increase would probably not say to his family: "Well, rounded to 5 significant figures, the pay increase I received is 3.3816% of my new pay level."

continuously, it will, at the end of the period, be equal to P_t, which is 25% higher than P_{t-1}.

The most common place continuous growth rates are encountered is in interest calculations. A banker could say: *We offer a per-annum interest rate, which, if rounded to 6 significant figures, is 22.3144%, and we compound continuously. At the end of a year, the principal will be 25% higher than it was at the beginning of the year.* If a year has 365 days and interest accumulation is approximated by compounding daily, the principal on day 2 would be: $Principal_{day\ 1} * (1 + \frac{\ln\left(\frac{25}{20}\right)}{365})$. Many banks also say: M*oney left on deposit from any day 0 to a subsequent day n will grow at this per-annum rate for n days.*

So, in cases where a quantification of the relative magnitude of an increase in price is desired, the Representative suggests focusing on the growth rate that, if applied to P_{t-1} and compounded continuously, leads at the end of the period to P_t.

How Does the Representative's Measure Apply to Postal Prices? The Case of a Product with One Price Element.

The justification provided by the Representative for using the log measure is that it "avoids calculating different percentage changes depending on which year is the base" (at 10).[2] In the case of the product with one price element that changed from 20 to 25, he is saying that he would prefer to avoid the alternatives of calling it a 25% increase or a 20% increase. Instead, he would take $\ln(25/20)$ and characterize the price as having a continuous growth rate of 22.3144%.

[2] Note that the periods need not be *years* and need not be of equal length.

When the Laspeyres formula, $\frac{P_t}{P_{t-1}} = \frac{\sum v_{t-1} * p_t}{\sum v_{t-1} * p_{t-1}}$, or, incidentally, the Paasche formula, $\frac{P_t}{P_{t-1}} = \frac{\sum v_t * p_t}{\sum v_t * p_{t-1}}$, is applied to a product with one price element, the right-hand-side reduces to $\frac{p_t}{p_{t-1}}$, which would become an input to my price index. Percentage increases are not an input, so the matter of selecting a base does not arise.[3] Nevertheless, in a further step, the Representative would take a log of this result and make the price index in period t equal to $RI_{t-1} * (1 + \ln\left(\frac{pt}{p(t-1)}\right))$. This would change the LHS of my central equality, and my proof would no longer hold. The Representative has not explained why his price index is superior, nor has he provided an interpretation of the new LHS. It is clear that his continuous growth rate, applied to p_{t-1}, would yield p_t, but it is not clear that using such growth rates helps arrive at a cost index. And another question that should be addressed is whether, to be consistent, any approach involving continuous growth rates should be extended to associated variables, such as factor price increases.

A degree of pause is also suggested by the fact that neither average prices for postal products nor associated price elements are continuous functions of time. In fact, they qualify fully as discontinuous. For example, prices changed at 12:01 a.m., April 17, 2011 and then again at 12:01 a.m., January 22, 2012, 280 days later. The April 17, 2011 prices did not begin immediately to grow at a rate that, if compounded continuously, would allow arrival at the new level at 12:01 a.m. on January 22, 2012. Instead, they were constant for 280 days, at which time a step increased occurred.

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[3] If desired, percentage increases can be calculated from the price relative. If the price in period t-1 is to be the base, the percentage increase $= 100 * (\frac{P_t}{P_{t-1}} - 1)$. If the price in period t is to be the base, the percentage increase $= 100 * (1 - 1/(\frac{P_t}{P_{t-1}}))$.

Also, when the prices of April 17, 2011 were implemented, the associated revenues increased immediately but did not then begin to grow at a continuous rate; rather, the effect on revenues, though influenced by volumes, was constant until a step increase on January 22, 2012.

How Does the Representative's Measure Apply to a Product with Multiple Price Elements?

For multi-price-element products, separate measures of $P_t \; and \; P_{t-1}$ do not exist. The Laspeyres formula weights elemental $p_t \; and \; p_{t-1}$ values to obtain an *estimate* of the $\frac{P_t}{P_{t-1}}$ relative, which goes into my price index. As in the single-price-element case, no percentage increases are calculated. Therefore, unless the Representative is suggesting an alternative to traditional index-number procedures, which I do not see in his work, he must be suggesting that logs be taken of the Laspeyres estimate of $\frac{P_t}{P_{t-1}}$. Neither the utility nor the implications of doing this have been explained.

Note that the index-number procedures can be viewed as providing values for the product-level $\frac{P_t}{P_{t-1}}$ relatives by applying revenue-share weights to the elemental $\frac{p_t}{p_{t-1}}$ relatives. Specifically, for a product with price elements A through K, the Laspeyres formula can be written $\frac{P_2}{P_1} = \frac{p_{A2}}{p_{A1}} * \frac{Rev_{A1}}{Tot\ Rev_1} + \frac{p_{B2}}{p_{B1}} * \frac{Rev_{B1}}{Tot\ Rev_1} + \cdots + \frac{p_{K2}}{p_{K1}} * \frac{Rev_{K1}}{Tot\ Rev_1}$, where Rev = revenue. Similarly, the Paasche formula can be written $\frac{P_2}{P_1} = \frac{1}{\frac{p_{A1}}{p_{A2}} * \frac{Rev_{A2}}{Tot\ Rev_2} + \frac{p_{B1}}{p_{B2}} * \frac{Rev_{B2}}{Tot\ Rev_2} + \cdots + \frac{p_{K1}}{p_{K2}} * \frac{Rev_{K2}}{Tot\ Rev_2}} \; or, \; inverting, \; \frac{P_1}{P_2} = \frac{p_{A1}}{p_{A2}} * \frac{Rev_{A2}}{Tot\ Rev_2} + \frac{p_{B1}}{p_{B2}} * \frac{Rev_{B2}}{Tot\ Rev_2} + \cdots + \frac{p_{K1}}{p_{K2}} * \frac{Rev_{K2}}{Tot\ Rev_2}$. I do not see that any of the Representative's concerns extend to the elemental $\frac{p_t}{p_{t-1}} \; or \; \frac{p_{t-1}}{p_t}$ relatives.

On the question of the Paasche index, though, keep in mind that: (1) It is not used in the price cap calculations. (2) It can be calculated after the fact only. (3) The understanding is basic that a Paasche index has no more claim to being "the actual" increase than does a Laspeyres index, although, under certain conditions, it can be argued that the Laspeyres index is an upper bound and the Paasche index is a lower bound. And, of some interest to economists, (4) using Laspeyres indexes with price caps provides better incentives to the agency to propose economically efficient prices.[4]

Indexing schemes have been a subject of attention in the literature for several hundred years. Developers of indexes generally, and proponents of specific indexes, focus on index properties and relations to other measures. The next tier of possibilities, after the Laspeyres and the Paasche indexes, would be the Marshall-Edgeworth index (based on average volume weights) and the Fisher index (the geometric mean of the Laspeyres and Paasche results). Both involve price relatives. I know of no indexes derived by taking logs of price relatives or final results.[5] And for use with price caps, only the Laspeyres index has properties relating to economic efficiency.

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[4] For a discussion of the efficiency properties of using Laspeyres indexes with price caps, with references to the literature, *see* Initial Comments of Time Warner Inc. in Response to Commission Order No. 26, September 24, 2007, pp. 6-10, Docket No. RM2007-1.

The intuition underlying the prescription that a Laspeyres cap will lead to more efficient prices than a Paasche cap is relatively simple. When a price element is increased, several things happen. First, the volume declines due to the own-price elasticity. Second, the volume declines due to any cross-price elasticities. Third, the costs change due to the volume changes. The Postal Service considers the effects of these changes on its bottom line and avoids spending its cap on inefficient increases for specific price elements (generally for relatively elastic price elements). The Paasche index, due to the volume decline, gives less weight to the increase for such elements, and thus allows the Postal Service to realize additional revenue without using as much cap.

[5] Of possible interest is the Törnqvist index (sometimes referred to as the Törnqvist-Theil index). It is the geometric average of the price relatives of each price element, with each weighted by the arithmetic average of its revenue shares in period 1 and period 2. Suppose there are two price elements and the relatives are 20 / 16 and 29 / 25. The first would be weighted by the average of its revenue shares in period 1 and period 2. The second would receive a corresponding weight. The Representative could

For present purposes, here is the bottom line. The Appendix in ACMA's Initial Comments showed that using Laspeyres indexes, along with the cost coverages, has an interpretation as a particular kind of cost index, specifically one consistent with a price-weighted quantity index. The Representative would replace the Laspeyres index with the logs of Laspeyres results, apparently. It is not clear what the interpretation would be of such a cost index, or why it would be superior. It is doubtful that the equations would boil down to something clear and meaningful, meaning that they might be uninterpretable.

Can Continuous Growth Rates Be Linked through Time?

As used here, indexes are useful for quantifying temporal behavior. It is common to peg an index at 100 in a base period and then, in a linking process, use period-over-period ratios to cover time. For example, the Laspeyres formula might show the average price in year 1 to be 5.3% higher than the average price in year 0. The price level in year 1 would be taken to be 100 * (1 + 0.053) = 105.3. Then if prices in year 2 were 6.3% higher than in year 1, the price level in year 2 would be (1 + 0.063) * 105.3 = 111.9339.

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consider replacing the relatives with their logs. I do not know what the properties of such an index would be. For price index formulas, *see*: http://en.wikipedia.org/wiki/List_of_price_index_formulas

An example covering four years of changes is shown in the following table.

Line	Measure	Year 0	Year 1	Year 2	Year 3	Year 4
1	Laspeyres ratio		1.053	1.063	1.046	1.083
2	Simple % increase		5.3%	6.3%	4.6%	8.3%
3	Conventional index	100	105.300	111.934	117.083	126.801
4	Log of Laspeyres ratio		0.05164	0.06110	0.04497	0.07973
5	Continuous growth rate		5.164%	6.110%	4.497%	7.973%
6	Index of logs, linked	100	105.164	111.589	116.608	125.906
7	Running Avg. of continuous growth rates		5.164%	5.637%	5.257%	5.936%
8	Avg. applied to principal of 100	100	105.300	11.934	117.083	126.801

As shown on line 3, a conventional price index shows the price level in year 4 to be 126.801, which means the prices, on average, are 26.801% higher than in year 0. Line 4 shows the logs of line 1, to yield the continuous growth rates on line 5. Linking these growth rates leads to the index in year 4 of 125.906, as shown on line 6. I have not found a way of interpreting this result.

Note that if a running arithmetic average of the line-5 growth rates is kept, as shown on line 7 (5.637% = (6.110% + 5.164%) / 2, 5.257% = (4.497% + 6.110% + 5.164%) / 3, and so on), a result can be obtained. If the price level of 100 in year 0 were allowed to grow at a per-annum rate of 5.936%, compounded continuously for 4 years, the price level in year 4 would be 126.801, exactly the result obtained on line 3. The formula is: $126.801 = 100 * e^{0.05936*4}$.

PART II – THE REPRESENTATIVE'S COMMENTS ON MY COST INDEX

The Representative's discussion of my cost index is divided between his text (at 10-13) and his Appendix (at 14-16), all pages numbered sequentially.

At the top of page 11, the Representative shows a perfectly proper formula for a cost index, for cost pools 1, 2, …, I, weighted by associated period-1 volumes (his equation 1).[6] Next, after explaining that obtaining the data required for that formula would be a "nontrivial" undertaking, he shifts to the alternative I suggest, characterized by the RHS of my proof (his equation 2), and asserts that I "incorrectly assert[ed] that [my RHS] represents a cost index" (at 12).

Since a total-cost index divided by a weighted quantity index is a fundamental formulation of a cost index, and since the RHS of my proof *is* a total-cost index divided by a weighted quantity index, the Representative's assertion is incorrect. He explains that the weighted quantity index implicit in my cost index "holds the rate constant in period two, not the unit cost, and allows the volume to vary" (at 12).[7]

The issue here is important. My cost index is equal in value to the quotient of a total-cost ratio divided by a corresponding *price*-weighted quantity index (my RHS, his equation 2), the latter using the same *prices* as weights in the numerator and denominator. Alternatively, a cost index could be obtained by dividing a total-cost ratio by a corresponding *cost*-weighted quantity index, the latter using the same *costs* as

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[6] To develop a cost index of the kind the Representative shows, cost pools must be selected. The pools often relate to specific productive operations, and could be disaggregated by plant. A difficulty encountered in postal operations is that different pieces going through the same operation sometimes have different costs. No need exists to align these pools with price elements. However, if the same costs are to be used in pricing, maybe to support discounts, there may be some economy in doing so. Another decision relates to whether to use period-1 weights or period-2 weights. Obviously, this is not an exact science.

[7] Consistent with prevailing postal parlance, I use the term "volume" to refer to piece volume, i.e., to the number of pieces. Accordingly, the term "volume index" would refer to the number of pieces in index form. But since, for example, processing and delivering pieces presented in 5-digit bundles is not the same amount of output as processing and delivering pieces presented in mixed-ACD bundles, the former requiring far less work, the piece volume does not contain much information about output. To refer to output, I use the term "quantity." But, since there is no common denominator for measuring quantity, index numbers must be used. Thus, a "quantity index," or, somewhat redundantly, a "weighted quantity index," would refer to the quantity of output in index form.

weights in the numerator and denominator. In both cases the volumes are allowed "to vary," and in both cases the weights in the numerator are the same as the weights in the denominator. So, it is not a matter of holding the "rate constant in period two" in one case and the "unit cost" constant in period two in the other, as the Representative states, but rather it is a matter of whether price weights or unit cost weights are used.[8] A further matter would be whether to use period-1 weights or period-2 weights.

If the Representative is saying nothing more than that a cost index derived from a *cost*-weighted quantity index might be more indicative than one consistent with a *price*-weighted quantity index, he has a point.[9] To that end, I have developed a cost-weighted index and show it in the text of these comments (*see* section V). It shows that my original cost index might be a slight understatement given the specific reality that was provided during the period of analysis.

But the Representative's exploration goes substantially beyond the question of whether a cost index derived from a *cost*-weighted quantity index would be a little higher or a little lower than my original one, as contained in his Appendix, to which I now turn.

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[8] In my ACR2011 Appendix, I pointed to the possibility of using marginal costs as weights (at 37). In its Reply Comments, Time Inc. pointed to the same possibility and went on to discuss associated implications (at 16).

[9] However, price-weighted quantity indexes may be more common than cost-weighted quantity indexes. First, it is a general presumption that prices in competitive markets tend toward marginal costs, so the two indexes are often very close to each other. Second, prices (and associated volumes) are usually more readily available than costs (and associated volumes). *See* billing determinants. Third, price information tends to be more unequivocal than cost information. And fourth, even when cost weights are used, the categories are often selected to align with the price elements.

The Representative's Appendix and His Exploration into an Alternative Way to Develop a Cost Index.

The central equality in my ACR2011 Appendix (my proof) is shown on the first page of this Appendix.[10] All of my computing was done with LHS variables. The RHS shows what the LHS is equivalent to, and thus provides its interpretation.

As an example (at 37-38), I presumed two RHS variables, a total-cost ratio of 1.15 and a price-weighted quantity index of 1.05, which, by definition, imply a cost index of 1.095 (1.15 / 1.05). I explained that in this situation, my LHS would yield a value of 1.095.

The Representative shows the RHS of my proof (using VI_2/VI_1, apparently, to represent the price-weighted quantity index) (at 14, his equation 3). He then recounts my example, which gives 1.095, and concludes that it would be "incorrect" to view 1.095 as a cost index (his equation 4, *et seq*.). Just as he did on page 12, he is rejecting as "incorrect" a basic formulation of a cost index. He does not argue that my LHS would not equal 1.095, which my Appendix showed. Neither does he argue the merits of an alternative quantity index.

Then, continuing in the same paragraph, as though to explain his rejection, but without addressing any matters relating to my cost index or to any index-number issue generally, he says: "Consider the definition of total mail processing cost for a firm with only one processing input: $TC = VxUC$" (at 14, his equation 5, a definitional identity). With this *request to consider*, the Representative departs from both the LHS and RHS of my central equality, as well as from any questions he has raised about whether to

[10] Interesting to note is that if a *Paasche* price index is used, the LHS of my proof would be equal in value to the cost index that would result from dividing the total-cost ratio by the price-weighted quantity index that would result from using *period-1* prices as weights.

rely on a price-weighted or a cost-weighted quantity index, and begins an exploration of his own into a way to assign responsibility for an increase in total cost to inherent changes in cost and, separately, volume. Nothing in my work hints that an exploration of the kind he pursues is relevant or likely to be productive, and no link allows the outcome of his exploration to have implications for my analysis. Thus his conclusion (at 16) that "the growth in unit costs cannot be separated from the growth in volume" applies only to his path, not to my work.

The following table illustrates the Representative's "firm with only one processing input." Here, from period 1 to period 2, the unit cost increases from 15 to 20 and the volume increases from 40 to 50.

Period	Period 1				Period 2		
Variable	U cost	Volume	T. cost		U cost	Volume	T. cost
Value	15	40	600		20	50	1,000
% change					33.33%	25%	66.67%
Log					0.287682	0.223144	0.510826
Continuous growth rt.					28.7682%	22.3144%	51.0826%

Note that no index-number problems exist. No changes in volume mix occurred. There is no difference between an index of piece volumes and a weighted quantity index. The cost increase is clear—an increase of 33.33 percent. No weighting scheme is needed.

The identity on total cost (his equation 5) is:

$$TC = V * UC \qquad \text{Eq. 1}$$

Adding an amount Δ to the volume and the unit cost yields:

$$TC + \Delta TC = (V + \Delta V) * (UC + \Delta UC) \qquad \text{Eq. 2}$$

Expanding:

$$TC + \Delta TC = V * UC + V * \Delta UC + UC * \Delta V + \Delta V * \Delta UC \quad \text{Eq. 3}$$

If ΔV and ΔUC are small, then ΔV * ΔUC is *doubly* small, and can be considered negligible, and can therefore be dropped. The first term on the RHS is V * UC, which is identical to TC. Therefore, if TC is subtracted from both sides of the equation, we have:

$$\Delta TC = V * \Delta UC + UC * \Delta V \quad \text{Eq. 4}$$

This equation, then, is an approximation (good only for small changes in V and UC).

If we divide both sides of this equation by TC, recognizing that TC ≡ V * UC, we obtain:

$$\frac{\Delta TC}{TC} = \frac{V * \Delta UC}{V * UC} + \frac{UC * \Delta V}{V + UC} \quad \text{Eq. 5}$$

Canceling and expressing in percentage terms yields:

$$\%\Delta TC = \%\Delta UC + \%\Delta V \quad \text{Eq. 6}$$

We have learned that for a "firm with one processing input," if the changes in unit cost and volume are small, the RHS reduces nicely and the percentage increase in unit cost (which is already known to be 33.33%) can be approximated by the percentage increase in total cost (66.67%) minus the percentage increase in volume (25%), which is 41.67%. If the changes were smaller, the approximation would be closer. Knowing this, however, does not add to our understanding of the changes that occurred, and it does not improve our quantification of them.

Now consider the Representative's exploration of subject identity (Eq. 1). He takes logs of both sides to obtain (his equation 7):

$$\ln(TC) = \ln(V) + \ln(UC) \quad \text{Eq. 7}$$

Taking a derivative with respect to time, he obtains (his equation 8a, viewing his equation 8 as composed of equations 8a, 8b, and 8c):

$$\frac{d\ln(TC)}{dt} = \frac{d\ln(V)}{dt} + \frac{d\ln(UC)}{dt} \qquad \text{Eq. 8}$$

Since, in general, d(lnZ) = dZ/Z, this equation can be written as (his equation 8b, except that he left an erroneous "ln" in each numerator):

$$\frac{dTC}{dt\,TC} = \frac{dV}{dt\,V} + \frac{dUC}{dt\,UC} \qquad \text{Eq. 9}$$

Recognizing that dZ/(dt Z) is a growth rate under continuous compounding, this equation can be written in terms of continuous growth rates (his equation 8c):

$$g_{TC} = g_V + g_{UC} \qquad \text{Eq. 10}$$

This is a reasonably interesting result, though not of clear usefulness. By expressing changes in the form of continuous growth rates, one can get away from an equation that is an approximation. Specifically, the continuous growth rate of unit cost (28.7682%) equals exactly the continuous growth rate of total cost (51.0826%) minus the continuous growth rate of volume (22.3144%). The equality holds for small changes and large changes.

Used in this way, natural logarithms have nice characteristics. But the answers were known before all this work was done. It was clear at the outset that the unit cost increased 33.33% (a continuous growth rate of 28.7682%), that volume increased 25% (a continuous growth rate of 22.3144%), and that "for a firm with only one processing input," there is no difference between the ratio of piece volumes and a weighted quantity index.[11]

[11] My proof was developed from the cost coverage (which recognizes all cost elements) and a price index (which recognizes all price elements), and there is no reason why the cost elements must be aligned with the price elements. Drawing on the formulation shown in the Introduction above, the implicit quantity index can be found easily by dividing the total-cost ratio by the LHS, and this can be done for the case of one price element or the case of many price elements.

The question is whether knowing a continuous growth rate is helpful. If, say, the price increases 12 %, the revenue will be the result of applying 112% of the old price to the new volume. The unit cost will be 33.33% higher, which, applied to the new volume, gives the new total cost. Thus, the cost coverage is affected directly by the 12% and the 33.33%. It is not clear what would be gained by expressing all of these variables in terms of continuous growth rates. The Representative could make a proposal along these lines.

Shifting to a Product with Two Volume Elements.

Next, the Representative requests that we "[c]onsider [the] total cost identity with the introduction of several processing steps" (at 15). From period 1 to period 2, the percentage increases in the unit costs and volume elements might be different from each other. For two volume elements, the Representative's cost identity (his equation 10) is:

$$TC = V_1 * UC_1 + V_2 * UC_2 \qquad \text{Eq. 11}$$

Adding an amount Δ to each variable, we obtain:

Eq. 10 can be rearranged to obtain: $g_{UC} = g_{TC} - g_V$. This is the Representative's equation 9, developed for the case of one volume element. g_V, in this case, is nothing more than the growth rate of piece volume—a weighted index is not needed and would not be different. g_{TC} is known, as it is in all cases. But, since g_{UC} is known already too, from the basic data, it is not clear why this equation is needed. And if it were used anyhow, both RHS variables are known.

Nevertheless, the Representative wants to use this equation to solve for g_{UC}. He does this in two steps. First, he solves my proof for my implicit quantity index (which in the case of one volume element is a simple ratio of piece volumes), takes a log to convert it to a continuous growth rate, g_V, and "emphasizes that the growth rate of volume is now identified" (at 15). Second, he puts g_V and g_{TC} into equation 9 and calls it a "method to identify the growth rate of mail processing unit costs" (at 15). This does not give a wrong answer, but the answer was known before any of the steps were taken. (Note that the unnumbered equation between the Representative's equations 8 and 9, at 15, is incorrect. A correct equation would be: $g_V = g_{TC} - g_\alpha$, where "g" indicates that a log has been taken.)

$$TC + \Delta TC = (V_1 + \Delta V_1) * (UC_1 + \Delta UC_1) + (V_2 + \Delta V_2) * (UC_2 + \Delta UC_2) \quad \text{Eq. 12}$$

This equation can be expanded, as before, and the second-order terms can be dropped, to obtain:

$$TC + \Delta TC = V_1 * UC_1 + UC_1 * \Delta V_1 + V_1 * \Delta UC_1 + V_2 * UC_2 + UC_2 * \Delta V_2 + V_2 * \Delta UC_2 \quad \text{Eq. 13}$$

Subtracting TC from both sides and dividing through by TC yields:

$$\frac{\Delta TC}{TC} = \frac{UC_1 * \Delta V_1 + V_1 * \Delta UC_1 + +UC_2 * \Delta V_2 + V_2 * \Delta UC_2}{V_1 * UC_1 + V_2 * UC_2} \quad \text{Eq. 14}$$

Nothing on the RHS cancels and it does not reduce to a simple expression involving percentage changes. And even if it did, there is no one ΔUC or %ΔUC to solve for. There is a ΔUC_1 and a ΔUC_2, and no way to put them together, a goal normally achieved by using index numbers. In effect, manipulating the total-cost identity does not help average the unit cost increases, and it would not even if the volume changes were all zero.

Not surprisingly, the Public Representative has similar difficulties. For two volume elements, each having a unit cost, his equation 10 (without the time subscripts) is identical to Eq. 11. He takes logs of both sides to obtain his equation 11a. For the two-element case, his equation 11a is:

$$\ln(TC) = \ln(V_1 * UC_1 + V_2 * UC_2) \quad \text{Eq. 15}$$

The log of a multiplicative product can be expressed as the sum of the logs of the multipliers and the multiplicands, a property used to obtain Eq. 7. But no such property exists for a sum, such as the sum in the parentheses on the RHS of Eq. 15.

Further, if a derivative with respect to time is taken, as was done to obtain Eq. 8, the result is (equivalent to his equation 11b, except that he erroneously left a "d" out of each numerator):

$$\frac{d\ln(TC)}{dt} = \frac{d\ln(V_1 * UC_1 + V_2 * UC_2)}{dt} \qquad \text{Eq. 16}$$

It is still true, generally, that d (lnZ) = dZ/Z, so Eq. 16 can be rewritten as (equivalent to his equation 11c):

$$\frac{dTC}{dt\ TC} = \frac{d(V_1 * UC_1 + V_2 * UC_2)}{dt\ (V_1 * UC_1 + V_2 * UC_2)} \qquad \text{Eq. 17}$$

The only continuous-growth-rate equation allowed is (his equation 11d):

$$g_{TC} = g_{V_1 * UC_1 + V_2 * UC_2} \qquad \text{Eq. 18}$$

This tells us that the continuous growth rate of the total cost is equal to the continuous growth rate of $V_1 * UC_1 + V_2 * UC_2$. But since $V_1 * UC_1 + V_2 * UC_2 \equiv TC$, we have learned only that (a) the continuous growth rate of the total cost is equal to the continuous growth rate of the total cost, (b) this growth rate cannot be expressed as the sum of the continuous growth rates of its components, and (c) it is not possible to solve this equation for the continuous growth rate of a single UC variable, basically because no single UC variable exists.[12]

The Representative concludes (at 16) that the "identification of a volume-constant unit-cost index is not possible." It is certainly true that taking logs and derivatives of a total-cost identity does nothing to average the unit costs in it, which means that his exploration failed. An indexing procedure is needed, and was supplied by my cost index. The Representative's "not possible" relates to his work, not mine.

[12] Though he could have, as he did for the single-volume-element case, the Representative does not point out here, for the multiple-volume-element case, that the quantity index can be obtained by dividing the total-cost ratio by my cost index.