BEFORE THE
POSTAL REGULATORY COMMISSION
WASHINGTON, DC 20268-0001

Consideration of Workshare Discount Rate Design	))	Docket No. RM2009-3

COMMENTS OF ROBERT W. MITCHELL ON MATTERS DISCUSSED AT THE PUBLIC FORUM AND IN EARLIER COMMENTS
(August 24, 2009)

Pursuant to Order No. 191, 13 comments were filed in this docket on May 26 and 27, 2009. On July 10, 2009, the Commission issued Order No. 243 ("Order on Further Procedural Steps"), which included provisions for a public forum (held August 11) and an invitation to file comments by August 31, 2009 "on the matters discussed at the public forum as well as the issues discussed in the [May-26-and-27] comments" (at 1 and 9). In response to that invitation, these comments represent my personal views.

A key issue is whether the constraints of Title 39, U.S.C. section 3622(e) apply to the relationship between single-piece and presorted First-Class Mail. In comments on this issue, a number of parties have pointed to the importance of delinking these categories and of the Postal Service having the flexibility to set rates for them in a way that maximizes its profits.[1]

[1] Maximizing profits is equivalent to maximizing net income, which can be done whether net income is negative or positive. Also equivalent is maximizing contribution.

The record will be more robust if it contains information on what the levels of profit-maximizing rates might be and on how far they are from the current rates. Also missing is a measure of the financial gain associated with one set of rates instead of another. My comments are a first cut at supplying this information. The analysis required is not easy, and some assumptions are needed. Others are invited to review my analysis and, in accordance with Commission preferences, to present an analysis of their own.

I. Comments Made by Parties

A number of observations have been rooted in the markups and relative contributions of single-piece and presorted First Class. Pitney Bowes said linking "frustrates efficiency because the Postal Service cannot use its pricing flexibility to encourage growth in more profitable Presort letter mail volumes."[2] Similarly, Bank of America *et al.* said that linking prevents the Postal Service from "us[ing] its pricing flexibility to stem declines and encourage growth in more profitable Presort letter mail volumes."[3] It also said "[a] 'linked' rate design will [ ] aggravate the existing disparity in profitability and contribution between Single-Piece and Presort letters . . . [and] accelerate the decline in mail volumes and thus [ ] increase the revenue shortfalls experienced by the Postal Service" (at 6).

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[2] Initial Comments of Pitney Bowes Inc., May 26, 2009, at 4.

[3] Initial Comments of the Bank of America Corporation, Discover Financial Services, J.P. Morgan Chase & Co., and The American Bankers Association, May 26, 2009, at 3.

The National Postal Policy Council (NPPC) said that "the Postal Service almost certainly could improve its financial position by substantially reducing presort First-Class rates (or increasing them at a below-average rate) and using the resulting headroom under the overall CPI cap [ ] to raise single-piece rates."[4] It added that "increasing rates on the more price-elastic product, presort First-Class Mail, while offsetting reductions in rates on the more price-inelastic product, single-piece mail, would reduce the net contribution from First-Class Mail as a whole" (at 5-6). It then noted that "[t]hese conclusions are corollaries of the standard economic formula for maximizing the overall profit of a multi-product firm, with or without an overall regulatory constraint on profits" (at 6). The Major Mailers Association supported the comments of NPPC (at 1). Observations similar to these were made at the public forum.

One is left with prescriptions that rate increases should always be lower for relatively profitable products, largely to encourage growth, and that lower increases for relatively elastic products are always the more profitable course. But these prescriptions invite questions. If relatively profitable products always receive lower increases, won't their relative profitability erode? Don't firms need their relatively profitable products? Should relatively elastic products always receive lower rate increases, or just until their markups get to a certain level?[5] In other words, where is

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[4] Comments of the National Postal Policy Council, May 26, 2009, at 5.

[5] On the question of rates for relatively elastic products, the National Postal Policy Council footnotes (at 6) a seminal article by William J Baumol and David Bradford ("Optimal Departures From Marginal Cost Pricing," 60 *Amer. Econ. Rev*. 265-283 (June 1970)). However, this article does not contain any statements that relate relative rate increases to relative elasticities. Rather, the article contains a formula for the appropriate levels of rates,

the stopping point? How does one tell when these prescriptions are making finances worse instead of better? Does the balance ever become just right? Where is the profit-maximum position? The current conditions of financial stress may add to arguments that profits should receive attention, but shouldn't the proponents of these arguments show some analysis?

Just as curious is that none of these commenters so much as mention the cross-elasticities, much less take them into consideration. Yet it is well known that cross-elasticities play a significant role in determining outcomes. The Postal Service's demand equations contain cross-elasticities for single-piece and presort First Class, as well as a cross-elasticity between presort First Class and Standard.[6] And in Order No. 243 the Commission encourages the provision of quantitative information about "smaller business mailers who can be converted (or already have been converted) by presort firms from users of single-piece into users of presorted First-Class" (at 5). It also asks for "information about how price signals influence mailers' decisions to invest in hardware, software or quality control processes" (at 5).

The importance of cross-elasticities should be apparent, particularly as they are central to gains emphasized by the oft-cited Efficient Component Pricing Rule (ECPR). If, in response to a discount that has just been increased, a single-piece mailer shifts

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given the elasticities. Once rates are set at the appropriate levels, the article's prescription is accomplished.

[6] The single-piece equation shows that a 10 percent increase in the discount would cause a 0.57 percent reduction in the volume of single-piece mail. Applying this percentage to the FY 2008 volume of 36.7 billion pieces suggests that 209 million pieces would shift to presort.

mail to a presort bureau (or decides it is time to invest in specialized mail preparation equipment), thereby moving to what tends to be regarded as a high-margin product (presort), though one that just received a relatively low rate increase, and if this shift lowers the Postal Service's revenues more than its costs, is it possible for the Postal Service's profits to increase? Can any of these matters be analyzed adequately without specific attention to the cross-elasticities? The answer is *no.*

II. Analysis with Cross-elasticities

If ratesetting is oblivious to the cross-elasticities, and they are influential, the goals pursued will not be achieved and the outcomes could be substantially different from those expected. The better course is to recognize the cross-elasticities and to build them into the analysis behind the rates.[7] That the analysis may be difficult, or that the relationships are not understood fully, is not a good excuse for not trying.

My analysis is contained, step by step, in an attached Appendix. The actual calculations are shown in my workpaper. The volumes (sums of the quarterly volumes for FY 2008), rates (including the presort discount, weighted averages for quarter 4 of FY 2008), and elasticities (own-price and cross) are taken from the Postal Service's demand analysis, transmitted by letter to the Commission on January 16, 2009.[8] The

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[7] For an analysis of the effects of cross-elasticities on economically efficient rate positions, *see*: Robert W. Mitchell, "Postal Worksharing: Welfare, Technical Efficiency, and Pareto Optimality," pp. 311-334, in *Emerging Competition In Postal And Delivery Services*, edited by Michael A. Crew and Paul R. Kleindorfer, Kluwer, Boston, 1999.

[8] *See* files DemandEquations-Nov2008.doc, pp. 1-21, and RCFDATA.xls, tab 1.

cost figures are for corresponding categories as contained in the Commission's FY 2008 CRA, LR-1-PRC, March 30, 2009. The cost of bulk metered mail (BMM) is taken from LR-3-PRC, March 30, 2009. Consistent with the rates proposed by the Postal Service on February 10 and implemented May 11, 2009, a cap of 3.770 percent is adopted.[9] The fact that only two categories (single-piece and presort) are considered makes the analysis rather aggregate, but demand equations are available for these two categories only.

An expression for the total contribution from First Class is maximized, yielding a set of maximum-profit rates for a period 2. The two key rate levels, single-piece and presort, both weighted averages, are tied through the rate-cap formula. In the base period, the average rate for single pieces is 52.5 cents, reflecting letters, flats, parcels, and non-machinable pieces, including pieces weighing more than one ounce. The average rate for presorted pieces is 36.54 cents, most being letters that weigh an ounce or less. The average presort discount is 8.84 cents. I assume that the average rate for pieces candidate to become presorted is 36.54 cents + 8.84 cents = 45.38 cents, and that the ratio of this rate to the average single-piece rate remains constant, a ratio I label as k_2. This assumption allows the average discount, which is equal to k_2 times the average single-piece rate minus the average presort rate, to change with the rates. The changes in volumes in going from the base rates to the maximum-contribution rates are the sums of the own-price and cross-price elasticity effects.

The Postal Service's equation for single-piece First Class contains a cross-

[9] Order No. 191, March 16, 2009, at 3.

elasticity to the average discount received by presorted pieces. This is slightly different from a cross-elasticity to the *price* of presort, but it is equivalent and is easy to work with. The equation for presort contains an instrument that is a fitted value of the discount deflated by a volume ratio, fitted because it is the result of regressing the deflated discount on other variables. I found that working with this instrument was more complex and appeared to suggest a volume shift to/from presort that is larger than the volume shift to/from single-piece. Since these two volume shifts must be equal, I decided to use the simpler single-piece shift and to apply it to both single-piece and presort.[10]

Once these relationships are built into the expression for the contribution, in such a way that it contains only one price, a partial derivative with respect to that price can be taken. Setting the partial derivative equal to zero and solving for the price gives the price that maximizes the contribution. Note that if presorted pieces revert and become single pieces, the cost they take on is the cost of bulk metered mail, not the average cost for all single pieces. A similar effect takes place if single pieces convert and become presorted pieces.

III. Results

For a cap of 3.770 percent, assuming the rate index is at the cap, results are shown in Table 1. Row 1 shows the profit-maximizing increases. The finding is that,

[10] If the cross-elasticities are larger than those included in my analysis, I presume their effect would be more pronounced. I see no reason why the qualitative nature of my findings would change.

based on the elasticities, cross-elasticities, costs, and volumes, an average rate increase of 4.75 percent for single-piece and 2.72 percent for presort, relative to the rates that existed in quarter 4 of FY 2008,[11] would have maximized the profits of the Postal Service. Based on FY 2008 volumes, these increases would have resulted in an increase in contribution from First Class of $1,192.7 million.

For comparison purposes, Row 2 shows the same information for the rate increases actually implemented on May 11, 2009, 3.97 percent for single-piece and 3.57 percent for presort. These rate increases, according to my model, are generating an increase in contribution of $1,191.1 million. The profit-maximizing increases (4.75% and 2.72%), then, would have generated just $1.6 million more in contribution than the rates actually implemented, which is about 0.023 percent of the $7.1 billion deficit now projected for FY 2009.

Another interesting reference point is an across-the-board increase, shown on Row 3. Such an increase would have increased the contribution by $1,190.2 million. Therefore, choosing the profit-maximizing increases (4.75% and 2.72%) instead of applying the cap equally to both categories would increase the Postal Service's profits by only $2.5 million. Note also that the profit-maximizing increases are no further than about 1 percentage point either way from the across-the-board increases, certainly closer than the reasoning of some commenters suggests.

If for a subsequent period, from the rate position resulting from the profit-

[11] The rates existing in quarter 4 of FY 2008 were those implemented on May 12, 2008. The rates used in the Postal Service's demand analysis, and here, are quarter-4 averages of those rates.

maximizing increases (4.75% and 2.72%), another profit-maximizing position were to be estimated, assuming unchanged elasticities, the rate increases for single-piece and presort would be expected to be much closer to each other.[12] There is no reason for one category to perpetually receive smaller or larger increases than another. That is, disparate increases may be needed to get to a profit-maximizing position, but once that position is reached, any disparity would be much less pronounced.

Row 4 is the same as Row 1, except that the cross-elasticities in the model are set equal to zero. That is, if there were no cross-elasticity between single-piece and presort, and if, accordingly, no cross-elasticity were put into the model, and the own-price elasticities remain as modeled, the profit-maximizing changes would be an *increase* in the single-piece rates of 10.15 percent and a *decrease* in the presort rates of 3.08 percent. The gain in contribution under this hypothetical would be $1,230.9 million, $38.2 million above the gain in Row 1. It is clear, then, that the effect of cross-elasticities existing, as they in fact do, is to constrain rather significantly what might be thought, mistakenly, to be the Postal Service's potential for increasing its profits. The cross-elasticities quantify market behavior that is influential in determining outcomes and the profit-maximizing position.

Note one more thing. If the rate changes implemented were actually to be an increase of 10.15 percent for single-piece and a decrease of 3.08 percent for presort,

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[12] *See* Antoinette Crowder and William C. Miller, "PA Dynamic Analysis of USPS Price Capped Standard Mail Rates Under Provisions of the Postal Accountability and Enhancement Act," presented at Advanced Workshop in Regulation and Competition, 27th Annual Eastern Conference, Skytop, PA, May 14-16, 2008, which maximizes welfare and finds that outcomes iterate toward a final equilibrium.

perhaps under the mistaken belief that the cross-elasticities are zero, the contribution actually realized would be $1,115.8 million, $115.1 million below the contribution of $1,230.9 million discussed above for the hypothetical Row 4, and $76.9 million below the profit-maximizing rates of Row 1. In other words, a significant loss relative to what might be expected, and relative to other available alternatives, would result from using a model that leaves out the cross-elasticities in an attempt to maximize profits.

Table 1

Average Rate Increases and Associated Contribution Increases for Single-piece and Presort First-Class Mail, under Varying Assumptions, from the Base Rates of Q4, FY 2008

		Col. 1	Col. 2	Col. 3
	Description	Single-piece	Presort	Increase in First-Class contribution due to rate changes, $ millions
Row 1	Profit-maximizing rate increases	4.75%	2.72%	1,192.7
Row 2	Rate increases implemented May, 2009	3.97%	3.57%	1,191.1
Row 3	Across-the-board increase	3.77%	3.77%	1,190.2
Row 4	Row 1 w/ cross-elasticities = zero	10.15%	-3.08%	1,230.9

Using the figures in my analysis, the only passthrough[13] that can be calculated is

[13] Generally, passthroughs are proportions of relevant cost differences that are passed through into rate differences, and are normally expressed in percentage terms. Here, the passthrough of interest is the proportion of the avoidance that is passed through into the presort discount.

somewhat non-traditional. Normally, the discount receiving the most attention is for presorted letters, and it is the difference between the rate for 1-ounce single pieces and the rate for 1-ounce mixed AADC automation pieces. To go with this, the cost difference is from the cost of bulk metered letters to the same AADC category. However, using the average discount in the Postal Service's demand analysis, I approximate the relevant single-piece rate at 45.3 cents, somewhat above the 1-ounce rate of 42 cents. This recognizes that some presorted pieces weigh over one ounce and some are flats. Also, the average discount in my analysis goes to an *average* presort piece, not to the tier for mixed AADC letters. Accordingly, the only option readily available is to compare my average discount to the difference between the cost of bulk metered mail and the average cost of presorted letters. This yields a passthrough percentage that is above the traditional one, but which is indicative.

So, let's consider the non-traditional passthrough described in the preceding paragraph. In the base position, the non-traditional passthrough is 145.1 percent, 32.9 percentage points above the passthrough found by the Commission for letters in its FY 2008 Compliance Determination (at 52). Under the profit-maximizing rates (Row 1), this measure increases to 164.2 percent, in small part because unchanged costs are being used and in larger part because the discount is allowed to increase with the rate differences. Thus, the passthrough under the profit-maximizing rates is higher than in the base. However, if the rate increases in Row 4 were to be implemented instead, based perhaps on a faulty analysis that fails to recognize the cross-elasticities, the passthrough would increase to 239.3 percent. Under profit maximization, then, it is

clear that the effect of recognizing the cross-elasticities is to reduce substantially the passthrough that is prescribed, potentially making it not far above (and possibly below) the ECPR passthrough.

IV. Conclusion

Under a cap, from a base position, an analysis should be possible to develop rates that would maximize the Postal Service's profits, recognizing both the own-price and cross-price elasticities. These comments provide such an analysis, consistent with the Postal Service's demand equations, starting from a base position of quarter 4 of FY 2008, just before the Postal Service proposed the rates it implemented on May 11, 2009. Nothing in my analysis supports prescriptions that certain categories should always receive relatively large or relatively small increases. The key findings are:

1. The own-price and cross-price elasticities can be recognized, and a profit-maximizing set of rates can be developed.

2. To maximize profits through rates to be implemented on May 11, 2009, increases for single-piece of 4.75 percent and for presort of 2.72 percent would have been needed. These increases are less than a percentage point different from the increases actually implemented, which were 3.97 percent and 3.57 percent, respectively.

3. Maximum-profit rates for May 11, 2009 would have yielded an additional contribution of $1.6 million relative to that of the rates actually implemented. Looking at another step, the rates actually implemented yielded an additional contribution of $0.9 million relative to that of across-the-board rates.

4. If the cross-elasticities are not recognized, and a model that is accordingly inapplicable is used to select "maximum-profit" rates, a rate *increase* for single-piece of 10.56 percent and a rate *decrease* for presort

of 2.73 percent would be indicated. These rate changes are substantially different from correct profit-maximizing changes. Also, due to the effect of the cross-elasticities, the contribution realized would be $115.1 million less than that shown by the inapplicable model.

5. The data available do not allow the profit-maximizing passthrough (of the presort cost avoidance into the presort discount) to be calculated, but it is clear that the presence of cross-elasticities makes the profit-maximizing passthrough substantially lower than it would be if the cross-elasticities were zero, perhaps putting it in the neighborhood of the ECPR passthrough.

To the extent that it is indicative, my analysis shows that the profit-maximizing rates for single-piece and presort First Class are not far from the current rates and that the financial gain to the Postal Service of moving to profit-maximizing rates would be small.

Appendix to Comments of Robert W. Mitchell, August 24, 2009

1. k_1 = active cap. If cap is 3%, k_1 is 1.03.
2. V = piece volumes, FY 2008, per USPS volume analysis.
3. R = rates, dollars per piece.
4. Cont = contribution, dollars.
5. C = cost, dollars per piece.
6. Subscripts: A = single-piece and B = presort.
7. Superscripts: 1 = base period and 2 = new period. Squarred variables always use parentheses and a superscript.
8. Thetas (θ) with subscripts are used for fixed values with multiple terms, to simplify expressions. They are defined as introduced.
9. e = own-price elasticity.
10. f = cross-price elasticity to the discount.
11. Equation numbers at right in parentheses, if numbered.
12. Delta (Δ) is used for a difference, thus $\Delta R = R^2 - R^1$.
13. Analysis tied to USPS demand analysis results, January 16, 2009.
14. Costs are taken from the FY 2008 CRA, LR-1-PRC, Compliance Determination, March 30, 2009.
15. The cost for bulk metered mail is taken from the Commission's FY 2008 Compliance Determination, LR-3-PRC.
16. V_A^1 = 36,715.610 million, V_B^1 = 49,162.964
17. R_A^1 = 0.525417 R_B^1 = 0.365401 weighted averages.
18. Average presort discount = 0.088375
19. C_A = 0.252 C_B = 0.112 C_{BMM} = 0.17389
20. Assume rate for pieces reverting to single-piece = 0.365401 + 0.088375 = 0.453776
21. Assume reverting pieces pay $k_2 R_A$ where k_2 = 0.453776/0.525417 = 0.863649
22. e_A = -0.217698 e_B = -0.250346
23. f_A = -0.057, cross elasticity of V_A to discount.

Assume rate ↑ = cap = k_1 $k_1 = 1.03$ for 3% cap

$$k_1 = \frac{V_A' R_A^2 + V_B' R_B^2}{V_A' R_A' + V_B' R_B'} = \frac{V_A' R_A^2 + V_B' R_B^2}{\Theta_1} \qquad (1)$$

Defines Θ_1

$$\therefore \Theta_1 k_1 = V_A' R_A^2 + V_B' R_B^2$$

and $$R_A^2 = \frac{\Theta_1 k_1}{V_A'} - \frac{V_B'}{V_A'} R_B^2$$

$$R_A^2 = \Theta_2 - V^* R_B^2 \qquad (2)$$

Defines Θ_2 & V^*

Allows replacing R_A^2 to allow solution in terms of R_B^2

Own-price elasticity relationships

$$e_A = \frac{\Delta V_A}{\Delta R_A}\frac{R_A'}{V_A'} \qquad (3)$$

$$\Delta V_A\Big|_{\text{due to own price}} = \frac{e_A V_A'}{R_A'}\Delta R_A = \frac{e_A V_A'}{R_A'}(R_A^2 - R_A')$$

Substituting Eq. 2 for R_A^2

$$\Delta V_A = \frac{e_A V_A'}{R_A'}(\theta_2 - V^* R_B^2 - R_A')$$

$$= \frac{e_A V_A'}{R_A'}(\theta_2 - R_A') - \frac{e_A V_A'}{R_A'}V^* R_B^2$$

$$= \theta_3 - \theta_4 R_B^2 \qquad \text{Defines } \theta_3 \text{ \& } \theta_4 \qquad (4)$$

$$e_B = \frac{\Delta V_B}{\Delta R_B}\frac{R_B'}{V_B'}$$

$$\Delta V_B = \frac{e_B V_B'}{R_B'}\Delta R_B = \frac{e_B V_B'}{R_B'}(R_B^2 - R_B')$$

$$= \frac{e_B V_B'}{R_B'}R_B^2 - e_B V_B'$$

$$= \theta_5 R_B^2 - \theta_6 \qquad (6)$$

Defines θ_5 & θ_6

Cross-price to discount elasticity

$$f_A = \frac{\Delta V_A}{\Delta(K_2 R_A - R_B)} \cdot \frac{(K_2 R_A' - R_B')}{V_A'} \qquad (7)$$

$$= \frac{\Delta V_A}{K_2 \Delta R_A - \Delta R_B} \cdot \frac{K_2 R_A' - R_B'}{V_A'}$$

$$\therefore \quad \frac{\Delta V_A}{K_2 \Delta R_A - \Delta R_B} = \frac{f_A V_A'}{K_2 R_A' - R_B'} = \theta_7 \qquad \text{Defines } \theta_7$$

$$\Delta V_A = \theta_7 (K_2 \Delta R_A - \Delta R_B) = \theta_7 (K_2 R_A^2 - K_2 R_A' - R_B^2 + R_B')$$

$$= \theta_7 (K_2 R_A^2 - R_B^2) + \theta_7 (R_B' - K_2 R_A')$$

Substitute Eq. 2 for R_A^2, get

$$\Delta V_A = \theta_7 \left(K_2 (\theta_2 - V^* R_B^2) - R_B^2 \right) + \theta_8 \qquad \text{Defines } \theta_8$$

$$= \theta_7 K_2 \theta_2 - \theta_7 K_2 V^* R_B^2 - \theta_7 R_B^2 + \theta_8$$

$$= \theta_7 K_2 \theta_2 + \theta_8 - \theta_7 R_B^2 (K_2 V^* + 1)$$

$$= \theta_9 - \theta_7 \theta_{10} R_B^2 \qquad (8)$$

Defines θ_9 & θ_{10}

Layering in volume adjustments from base position

$$Cont^2 = V_A' R_A^2 + \Delta V_A \mid R_A^2 + \Delta V_A \mid K_2 R_A^2$$

(R_A^2: Eq.2; own price: Eq. 4; $K_2 R_A^2$: Eq.2; discount: Eq. 8)

$$+ V_B' R_B^2 + \Delta V_B \mid R_B^2 + \Delta V_B \mid R_B^2$$

(R_B^2: own price Eq. 6; R_B^2: discount - Eq. 8)

$$- \left[V_A' C_A + \Delta V_A \mid C_A + \Delta V_A \mid C_{BMM} \right]$$

(C_A: own price Eq. 4; C_{BMM}: discount Eq. 8)

$$- \left[V_B' C_B + \Delta V_B \mid C_B + \Delta V_B \mid C_B \right]$$

(C_B: own price Eq. 6; C_B: discount - Eq. 8)

Substituting equations into contribution expression

$$Cont^2 = V_A'(\theta_2 - V^* R_B^2) + (\theta_3 - \theta_4 R_B^2)(\theta_2 - V^* R_B^2)$$

$$+ k_2 (\theta_9 - \theta_7 \theta_{10} R_B^2)(\theta_2 - V^* R_B^2)$$

$$+ V_B' R_B^2 + (\theta_5 R_B^2 - \theta_6) R_B^2$$

$$- (\theta_9 - \theta_7 \theta_{10} R_B^2) R_B^2$$

$$- \left[V_A' C_A + (\theta_3 - \theta_4 R_B^2) C_A \right.$$

$$\left. + (\theta_9 - \theta_7 \theta_{10} R_B^2) C_{BMM} \right]$$

$$- \left[V_B' C_B + (\theta_5 R_B^2 - \theta_6) C_B \right.$$

$$\left. - (\theta_9 - \theta_7 \theta_{10} R_B^2) C_B \right]$$

Multiplying Out

$$
\begin{aligned}
Cont^2 = {} & \theta_2 V_A' - V_A' V^* R_B^2 + \theta_2 \theta_3 - \theta_3 V^* R_B^2 \\
& - \theta_4 \theta_2 R_B^2 + \theta_4 V^* (R_B^2)^2 + k_2 \theta_9 \theta_2 \\
& - k_2 \theta_9 V^* R_B^2 - k_2 \theta_2 \theta_7 \theta_{10} R_B^2 \\
& + k_2 \theta_7 \theta_{10} V^* (R_B^2)^2 \\
& + V_B' R_B^2 + \theta_5 (R_B^2)^2 - \theta_6 R_B^2 - \theta_9 R_B^2 \\
& + \theta_7 \theta_{10} (R_B^2)^2 \\
& - V_A' C_A - \theta_3 C_A + \theta_4 C_A R_B^2 - \theta_9 C_{BMM} \\
& + \theta_7 \theta_{10} C_{BMM} R_B^2 \\
& - V_B' C_B - \theta_5 C_B R_B^2 + \theta_6 C_B \\
& + \theta_9 C_B - \theta_7 \theta_{10} C_B R_B^2
\end{aligned}
$$

Maximization

Take partial derivative w.r.t. R_B^2

set = 0

Move R_B^2 terms to RHS, obtain

$$-V_A' V^* - \theta_3 V^* - \theta_4 \theta_2 - k_2 \theta_9 V^* - k_2 \theta_2 \theta_7 \theta_{10}$$

$$+ V_B' - \theta_6 - \theta_9 + \theta_4 C_A + \theta_7 \theta_{10} C_{BMM}$$

$$- \theta_5 C_B - \theta_7 \theta_{10} C_B$$

$$= -2\left[\theta_4 V^* + k_2 \theta_7 \theta_{10} V^* + \theta_5 + \theta_7 \theta_{10}\right] R_B^2$$

$$\therefore \quad R_B^2 = \frac{LHS}{-2[\quad]}$$

Given R_B^2, R_A^2 is found from Eq. 2.