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BEFORE THE
POSTAL REGULATORY COMMISSION
WASHINGTON, D.C. 20268-0001

REGULATIONS ESTABLISHING SYSTEM OF RATEMAKING	:	**Docket No. RM2007-1**

STATEMENT OF
ANTOINETTE CROWDER AND WILLIAM C. MILLER
IN RESPONSE TO COMMISSION ORDER NO. 26
ON PROPOSED RULES ESTABLISHING A SYSTEM OF RATEMAKING

INTRODUCTION

We are consultants in regulated industry economics and each of us has more than twenty years experience in postal policy and ratemaking issues, working with a variety of mailers on numerous issues. We have both been involved in all the omnibus rate cases since R87-1 and have also been involved in many of the mail classification and rulemaking dockets as well. Over many years, we have worked on postal reform issues and, thus, have followed with great interest the progress of this rulemaking. We commend the Commission on its proposed rules. As we see it, they follow closely the intent of the Postal Accountability and Enhancement Act (PAEA).

In this set of comments, we focus very narrowly on the Commission's rules regarding annual and partial-year average class price limitations. In particular, we are concerned about the Section 3100.22 partial-year limitation. We agree that the Postal Service should have access to partial-year rate increase authority, under specific conditions. However, we do not agree with the proposed way in which that partial-year increase would be calculated. The partial-year increase calculation, as proposed, would always increase average class prices more than the inflation rate would warrant. And, depending upon how the subsequent postal prices were developed after a partial-year price increase was granted, there is the distinct possibility that average class price increases would continue to grow far faster than cumulative inflation over the same time period. This situation is contrary to the PAEA and we believe the Commission did not intend this result.

Accordingly, in this statement, we use the Commission's Examples 1 and 2 to demonstrate that the Commission's proposed partial-year adjustment would cause price increases greater than inflation. We also provide a technical discussion of why it occurs, proof that it always causes price increases greater than inflation, and show how a better partial-year adjustment can be designed and implemented.

Separately, we also make a recommendation on how to calculate an annual inflation percentage increase that is statistically superior to the one that would be calculated under proposed rules in Sections 3100.21 and 3100.22.

We have developed additional technical documentation supporting these issues and are willing to provide it, upon request. We also would be willing to participate in a technical conference if one is requested.

I. The Proposed Section 3100.22 Partial-Year Limitation

The Commission's proposed Section 3100.22 partial-year formula for adjusting average postal prices within a rate year is incorrect and will always cause average prices to increase in excess of the most recent annual CPI-U rate. Using the Commission's Example 2 (page 30), we show that the excess is due to counting the percent CPI-U change applicable to the first six months of the rate year twice: once when the annual adjustment is made and again in the partial-year adjustment. In order to understand this problem, a review of the rate cap concept is helpful.

(a) The Rate Cap Concept And The Commission's Incorrect Partial-Year Adjustment

The rate cap concept, under the PAEA, limits cumulative percentage price increases to the cumulative percentage CPI-U increases. By permitting annual price increases up to the estimated increase in annual inflation, rate capping provides the Service with average price growth sufficient to cover inflationary growth in the prices of postal resources with the intent of providing the Service with a reasonable opportunity to remain financially viable. Moreover, the cap also provides the incentive for the Service to improve operational and pricing efficiency so that it can earn profits.

Establishment of rate caps begins with the assumption that rates for the year prior to the first rate cap year are set so that they generate sufficient annual revenue to cover the estimated annual costs for the same year. Those estimated annual costs include all estimated impacts from inflation experienced during that year. For simplicity in explanation, the year prior to the first rate cap year is Year (t-1) and the first rate cap year is called Year (t).

Thus, for the first year of the rate cap process (Year (t)), the rate cap limitation permits the average price for Year (t) to increase over that for Year (t-1) only to the extent that inflation (CPI-U) for Year (t) is *estimated* to increase (percentage-wise). At the beginning of Year (t), the average annual price for Rate Year (t) is P_t, developed as:

$$P_t = P_{(t-1)} * CPI_{(t-1)}/CPI_{(t-2)}$$

Where:

$CPI_{(t-1)}$ = Actual average monthly CPI-U for Year (t-1)
$CPI_{(t-2)}$ = Actual average monthly CPI-U for Year (t-2)
$CPI_{(t-1)}/CPI(t-2)$ = *Actual* inflation increase factor for Rate Year (t-1)
= *Estimated* inflation increase factor for Rate Year (t).

Thus, the percentage increase from $P_{(t-1)}$ to P_t equals the estimated percentage increase in postal resource prices over Year (t). If the *actual* inflation experienced over Rate Year (t) is the same as the *estimated* inflation used to develop the average price at the beginning of Year (t), then the rate cap has worked as intended.

However, if actual experience during Rate Year (t) shows that the beginning-of-year estimate of the inflation increase for Rate Year (t) is too low, then the Commission has determined that it is appropriate to make a partial-year price adjustment. In order to ensure that this partial-year price adjustment does not increase rates more than the preceding 12-month percentage change in inflation, the adjustment should reflect the difference between (1) the estimate made at the beginning of the year and (2) the actual experience during the year. We believe such an adjustment is appropriate but that the Commission's proposed calculation is incorrect and results in an excessive increase.[1]

(b) Example Demonstrating The Incorrect Partial-Year Adjustment

The problem with the Commission's proposed calculation can be demonstrated by using the Commission's Example 1 and 2 data (reproduced in Figure 1 below). Assume the first capped annual average price is in April 2006 and that the annual average price immediately before rate cap implementation is 10¢. As the Commission's Example 1 shows, the estimated inflation rate for the April 2006-March 2007 period is 3.5%:

$$3.5\% = 197.0/190.3 - 1 = CPI_{(t-1)}/CPI_{(t-2)} – 1.$$

So, the annual average price of 10¢ in Year (t-1) can be increased to 10.35¢ in Year (t). The 3.5% inflation rate estimated for Year (t) is already reflected in the 10.35¢ price.

Now, in September of Year (t), the Service requests a partial-year increase to begin in October, per the Commission's proposed calculation. As the Commission's Example 2 shows, the partial-year rate increase is 1.8%:

$$1.8\% = 200.6/197.0 -1 = \text{Recent 12-month } CPI/CPI_{(t-1)}.$$

[1] One could argue that a partial-year adjustment is not appropriate at any time because, over time, the difference between estimated and actual annual inflation rates should balance out. Moreover, the PAEA rate cap process does not guarantee annual breakeven/surplus financial conditions for the USPS but only provides a reasonable opportunity for the Service to succeed over the longer term. However, we believe that the Commission should consider the Service's current financial state and include provision for the Service to quickly change its rates if they are based on an estimated inflation rate that turns out to be too low.

In total then, the new October 2006 price is 10.54¢ = 10.35¢ * 1.018. However, that price is 5.4% greater than the October 2005 (through March 2006) price of 10.0¢ while the immediately preceding 12-month average CPI-U increase (between October 2005 and September 2006) is only 3.7%:

$$3.7\% = 200.6/193.5 = CPI_{(October2005\text{-}September2006)}/CPI_{(October\ 2004\text{-}September\ 2005)}.$$

FIGURE 1 – COMMISSION EXAMPLES 1 AND 2 CPI-U DATA

Month	Actual CPI	Average Price	12-Month ACPI-U	
April 2004	188.0			
May	189.1			
June	189.7			
July	189.4			
August	189.5			
September	189.9			
October	190.9			
November	191.0			
December	190.2			
January	190.7			
February	191.8			
March	193.3			
April 2005	194.6	0.100		
May	194.4	0.100		
June	194.5	0.100		
July	195.4	0.100		
August	196.4	0.100		
September	198.8	0.100	193.500	(n) in
October	199.2	0.100		Calculations
November	197.6	0.100		
December	196.8	0.100		
January	198.3	0.100		
February	198.7	0.100		
March	199.8	0.100		
April 2006	201.5	0.104		
May	202.5	0.104		
June	202.9	0.104		
July	203.5	0.104		
August	203.9	0.104		
September	202.9	0.104	200.633	(g) in
October		0.105		Calculations

CALCULATIONS:		
Assume average class price (Pt-1) for April 2005-March 2006 is:	$0.100	(a)
Example 1 April 2006 Annual Price Increase		
Average CPI-U April 2005-March 2006	197.0	(b)
Average CPI-U April 2004-March 2005	190.3	(c)
April 2006 Price Increase	3.5%	(d) = (b)/(c) - 1
New Pt price in April 2006	$0.1035	(e) = (a) *(1+ (d))
Example 2 October 2006 Partial-Year Price Increase		
Average CPI-U April 2005-March 2006	197.0	(f) = (b)
Average CPI-U Oct 2005-Sept 2006	200.6	(g)
October 2006 Price Increase	1.8%	(h) = (g)/(f) - 1
New Pt* price in October 2006	$0.1054	(I) = (a) * (1+(d)) * (1+(h)) = (e) * (1+(h))
October 2005-October 2006 Price Increase	5.4%	(j) = (I)/(a) - 1
Average Year(t) Price	$0.1045	(k) = 6/12*(e)+6/12*(I)
Average Year(t) Price/Average Year (t-1) price	4.5%	(l) = (k)/(a)
Average CPI-U Oct 2005-Sept 2006	200.6	(m) = (g)
Average CPI-U Oct 2004-Sept 2005	193.5	(n)
October 2005-September 2006 Avg CPI-U Inc	3.7%	(o) = (m)/(n) - 1

Moreover, the average price (P_t*) for the Rate Year (t) after the partial year adjustment in October is 4.5%:

$P_{(t-1)}$ = 10¢ and

$P_t^* = 6/12 * P_{toriginal} + 6/12 * P_{tpartial}$

P_t^* = 6/12 * 10.35¢ + 6/12 * 10.54¢ = 10.45¢, and

$P_t^* / P_{(t-1)} - 1$ = 10.45¢/10¢ - 1 = 4.5%..

This 4.5% average Rate Year (t) price increase should be compared to the 3.7% actual inflation increase for the most recent 12-month period available.

Thus, the proposed partial-year adjustment results in a partial-year rate increase (5.4%) and resulting average annual rate increase (4.5%) that are both greater than the corresponding inflation increase (3.7%) for the most recent 12-month period. Moreover, if the inflation estimate for the next rate year (Year (t+1)) is then used to increase either

the partial-year price (10.54¢) or the average year price (10.45¢) to produce $P_{(t+1)}$, then that average price will also be excessive. Such a situation would cause the average price in succeeding rate years to always be higher than what would be allowed under the Commission's proposed annual rate increase authority.

(c) Mathematical Demonstration Of The Incorrect Partial-Year Adjustment

To understand the problem with the proposed partial-year adjustment, we first look at the Commission's correct annual price adjustment:

Annual Limitation for Postal Price in Rate Year (t) = $CPI_{(t-1)} / CPI_{(t-2)} - 1$.

Then the average price for Rate Year (t) = P_t and:

$P_t = P_{(t-1)} * [CPI_{(t-1)} / CPI_{(t-2)}]$.

This annual limitation (converted to a percentage by multiplying by 100) is simply the estimated percent increase in average monthly CPI-U for Year (t), made at the beginning of Year (t), based on the actual experience in Year (t-1), as shown below:

$CPI_{test} = CPI_{(t-1)} * CPI_{(t-1)} / CPI_{(t-2)}$, and

$CPI_{test} / CPI_{(t-1)} = CPI_{(t-1)} / CPI_{(t-2)}$.

Where:

CPI_{test} = Estimated average monthly CPI-U for Rate Year (t)
$CPI_{(t-1)}$ = Actual average monthly CPI-U for Year (t-1)
$CPI_{(t-2)}$ = Actual average monthly CPI-U for Year (t-2).

Thus, the average postal price for Year (t) can increase no more than the estimated average CPI-U percent increase for the same year. Also, note that each of the average year CPI-U figures can be substituted by the sum of the weighted average monthly CPI-Us for the first and last six months of the same year. So, in the Commission's example:

$CPI_{(t-1)} = (CPI_{(t-1)1} * 6/12) + (CPI_{(t-1)2} * 6/12)$.

Where:

$CPI_{(t-1)1}$ = Actual CPI-U average for the first 6 months of rate year (t-1)
$CPI_{(t-1)2}$ = Actual CPI-U average for the second 6 months of rate year (t-1).[2]

[2] The two periods need not be six months. They can be any combination of months summing to 12. In that more general case, the weight applied to the average CPI-U for a particular number of months is: number of months represented/12. We can provide, upon request, documentation on how this can be done.

And, finally, the annual price limitation can be restated by dividing the last by the equivalent representation for $CPI_{(t-1)}$ and subtracting one:

$$CPI_{(t-1)} / CPI_{(t-2)} - 1 = [(CPI_{(t-1)1} * 6/12) + (CPI_{(t-1)2}) * 6/12] / [(CPI_{(t-2)1} * 6/12) + (CPI_{(t-2)2} * 6/12)] - 1$$

All of this is as it should be.

With respect to the partial-year price adjustment, then, the intent is to adjust the Year (t) average price when the rate of inflation actually experienced in that year exceeds the beginning-year estimate (estimated as being the same as that experienced in Year (t-1)). This ensures that unexpected increases in the inflation rate, which are beyond the Service's control, cannot unduly harm the Service's financial position. Unfortunately, the Commission's partial-year price adjustment proposal is incorrect. That proposed adjustment is:

Partial-Year Limitation in Rate Year (t) $= (CPI_{t1} + CPI_{(t-1)2}) \ / \ (CPI_{(t-1)1} + CPI_{(t-1)2}) - 1$

Where:

CPI_{t1} = Actual average monthly CPI-U for the first 6 months of Rate Year (t).

Thus,

$$P_{tpartial} = P_t * (1+\text{partial year limit}) = P_{(t-1)} *(1+ \text{annual limit}) * (1+ \text{partial year limit}).$$

$$= P_{(t-1)} * (CPI_{t1} + CPI_{(t-1)2}) \ / \ (CPI_{(t-2)1} + CPI_{(t-2)2})$$

This demonstrates the problem. $P_{tpartial}$ is simply $P_{(t-1)}$ multiplied by an 18-month CPI-U change (i.e., from the beginning of Year (t-2) to the end of the first six months of Year (t)). This is excessive since the $P_{tpartial}$ limitation should be based only on the difference between the 12-month average monthly CPI-U rate of change used to establish P_t and a more recent 12-month average monthly CPI-U rate of change. If, during the rate year (t) the CPI-U rate of change remains constant (or declines), then there should be no need for a partial-year adjustment.

Unfortunately, the Commission's proposed partial year adjustment always permits a partial-year increase, whether there is a need for one or not. This is because P_t reflects the estimated 12-month average CPI-U percentage increase while the partial-year adjustment adds an additional 6-month CPI-U change. One way to look at this is to see that P_t was developed using an *estimated* CPI-U annual rate for the first six months of Year (t) and $P_{tpartial}$ is developed using the *actual* annual CPI-U rate for the same six months. This "double-count" can be evidenced assuming a constant inflation rate. First, reorganize the Commission's partial-year limitation into the following:

$$(CPI_{t1} + CPI_{(t-1)2}) \ / \ (CPI_{(t-1)1} + CPI_{(t-1)2}) = (CPI_{t1}/CPI_{(t-1)1})*w_{1t} + (1 - w_{1t}),$$

Where:

$w_{1t} = CPI_{(t-1)1}/(CPI_{(t-1)1} + CPI_{(t-1)2})$
$1 - w_{1t} = CPI_{(t-1)2}/(CPI_{(t-1)1} + CPI_{(t-1)2})$.

Then suppose there is a constant inflation rate $r > 0$ so that $1 + r = CPI_{(t-1)}/CPI_{(t-2)} = CPI_{t1}/CPI_{(t-1)1}$. In that case, substitution gives:

$$(CPI_{t1}/CPI_{(t-1)1}) * w_{1t} + (1 - w_{1t}) = (1 + r) * w_{1t} + (1 - w_{1t}),$$

and the partial-year limit proposed by the Commission is:

$$\text{Partial Year Limitation in Rate Year (t)} = (CPI_{t1} + CPI_{(t-1)2}) \ / \ (CPI_{(t-1)1} + CPI_{(t-1)2}) - 1$$

$$= r * w_{1t}.$$

Then using the formula for $P_{tpartial}$ above, the adjusted rate when the inflation rate is constant can be shown as:

$$P_{tpartial} = P_{(t-1)} * (1+r) * (1+r * w_{1t}).$$

This clearly shows that the Service could seek a mid-year windfall equal to $P_{(t-1)} * (1+r) * r * w_{1t}$ even though the inflation rate remains as expected at the start of Rate Year (t). In other words, the existing inflation rate has been double-counted: once when setting the rate at the start of the year, and then once more through the last multiplicative term when setting the adjusted rate. This can most easily be seen in a simplified situation with a constant inflation rate – e.g., a constant 3.5% rolling average over Year (t-1) that continues through Year (t). Then at the beginning of Year (t), average price is correctly set as follows:

$$P_t = P_{(t-1)} * 1.035.$$

But, under the Commission's proposed partial-year adjustment, the Service can then increase P_t during Year (t) – e.g., mid-year:

$$P_{tpartial} = P_{(t-1)} * 1.035 * (1+.035 *0.496) = P_{(t-1)} * 1.053$$

Where $0.496 = w_{1t} = CPI_{(t-1)1}/(CPI_{(t-1)1} + CPI_{(t-1)2})$.[3]

[3] For illustrative purposes and to simplify, we use the Commission's CPI-U data to estimate the value of w_{1t} under a constant inflation rate.

And, therefore average price over the entirety of Year (t) is:

$$P^* = P_{(t-1)} * 1.035 * 6/12 + P_{(t-1)} * 1.053 * 6/12 = P_{(t-1)} * 1.044.$$

Through the incorrect partial-year adjustment, the Service has acquired a 4.4% average price increase for Year (t) even though there has been no change in the annual 3.5% rate of inflation. (This also explains the result in the Part I(b) example above, using the Commission's CPI-U data.)[4]

Thus, the Commission's partial-year adjustment will always incorrectly result in a price increase greater than warranted, regardless of whether there are constant or varying rolling average inflation rates since the one estimated at the beginning of the rate year.[5] Moreover, if that partial-year price is then used as the basis for the subsequent year (t+1)'s average price, then clearly $P_{(t+1)}$ will also be excessive. Clearly, continuation of successive annual and partial-year price adjustments will cause the average price over time to increase much faster than cumulative inflation.

(d) Correction of the "Partial Year Limitation"

An appropriate "partial year" limitation can be derived from the above analysis. As indicated before, the rate increase limitation at the start of Rate Year (t) implies an average CPI-U estimate for the year:

$$CPI_{test} = CPI_{(t-1)} * (CPI_{(t-1)} / CPI_{(t-2)}).$$

For the Commission's Example 2, $CPI_{test} = CPI_{t1est} * 6/12 + CPI_{t2est} * 6/12$,

where:

$$CPI_{t1est} = CPI_{(t-1)1} * (CPI_{(t-1)} / CPI_{(t-2)})$$
$$CPI_{t2est} = CPI_{(t-1)2} * (CPI_{(t-1)} / CPI_{(t-2)}).$$

In the Commission's Example 2, as Rate Year (t) proceeds, actual monthly CPI-U data from the first six months of Year (t) can be used to update the Year (t) estimate

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[4] As noted in footnote 2 above, using the Commission's Example 1 and 2 data, the estimated inflation increase at the beginning of Year (t) was 3.5% while the actual 12-month increase at the end of September was a slightly higher 3.7%. Accordingly, the Service would be entitled to some partial-year increase. Using the correct partial-year adjustment (described below), the Service would be entitled to a mere 0.13% increase on top of the 3.5% it had already received at the beginning of Year (t). Any additional partial-year increase beyond that would simply be a windfall for the Service.

[5] Note that, as long as inflation increases (regardless of the rate of increase), the rolling 12-month average monthly CPI-U will increase. This guarantees the Commission's "partial year" approach – a more recent 12-month CPI-U average divided by an earlier 12-month average – will cause an excessive price increase. It can be proved generally that the Commission's approach will always increase rates more than the correct approach.

of average monthly CPI-U – update CPI_{test} to $CPIN_{test}$. This updated (new) average monthly CPI-U estimate for Rate Year (t) is:

$$CPIN_{test} = CPI_{t1} * 6/12 + CPIN_{t2est} * 6/12$$

where:

CPI_{t1} = Actual average monthly CPI-U for the first 6 months of Rate Year (t)
$CPIN_{t2est} = CPI_{(t-1)2} * [(CPI_{t1} + CPI_{(t-1)2})/(CPI_{(t-1)1} + CPI_{(t-2)2})]$.

Note that the updated average CPI-U estimate for the second half of Year (t) is based on the updated estimate of the inflation rate derived from the last twenty-four months of data (the bracketed expression). Using the latest actual 12-month monthly CPI-U percentage increase is perfectly consistent with the Commission's correct approach to estimating the annual inflation for Year (t) at the beginning of that year. In the Commission's example, this CPI-U rate of increase is the 3.7% cited earlier.

Thus, the actual data from Year (t) is used to replace: (1) the beginning-of-year estimated average monthly CPI-U for the first half of the year with the actual average monthly CPI-U for the same period and (2) the beginning-of-year estimated average monthly CPI-U for the second half of the year with an updated CPI estimate. Accordingly, an updated mid-year price (PN_t) – the correct partial-year average price for the last 6 months of Year (t) – can be calculated as:[6]

$$PN_t = P_{(t-1)} * CPIN_{test} / CPI_{(t-1)}$$

$$= P_{(t-1)} * CPI_{test}/CPI_{(t-1)} * CPIN_{test} / CPI_{test}$$

$$= P_t * CPIN_{test} / CPI_{test}.$$

Clearly a mid-year change from P_t would be justified only if the new average monthly CPI-U estimate for Year (t) were greater than the old, i.e. $CPIN_{test} / CPI_{test} > 1$. If the inflation rate were unchanged, then $CPIN_{test} = CPI_{test}$ and therefore $PN_t = P_t$. No rate adjustment would be justified because the inflation rate has not changed.[7]

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[6] This assumes that the "less-than-annual" or "partial year" limitation means that the rate derived from the procedure lasts only through the remainder of the current rate year. However, we can also envision a rate change occurring sometime during a rate year that could last beyond that rate year – for example if the Postal Service wished to change the schedule of annual rate adjustments. It is not clear whether the Commission's rules provide such an opportunity but the general approach proposed here could easily be adapted to accommodate that situation.

[7] This adjustment permits the Service to cover the greater inflation impact from the second half of Rate Year (t) but does not allow it to recover any from the first half of the same year. Clearly the Commission intends to provide the Service with an opportunity to increase rates more quickly in times of increasing inflation rates. However, there is no comparable adjustment to reduce rates in times of declining inflation rates. We believe that it is unlikely, once the Section 3100.22 calculation has been corrected and administrative process has been clarified, that the Service will use this partial-year adjustment very often,

This demonstration can be generalized to cover any number of months into the rate year rather than just the Commission's example of 6 months into the rate year. It can also be extended to cover the situation where the "partial-year" adjustment price (PN_t) is intended to remain in place through some months of subsequent Year (t+1), should the Service wish to do that.

(e) Annual Average Price Change Following A Year With A Partial Year Adjustment

From the above, it follows that the annual average price limitation for the next year (Year (t+1)) should reflect the fact that there was a "partial-year" price adjustment in Year (5). As already discussed, the Commission's annual limitation for Year (t+1) uses the actual inflation rate for the most recent 12 months – i.e., the actual inflation rate for Year (t). However, the Year (t) partial-year price of PN_t already reflects the inflation experienced in the first six months of Year (t). Thus, it would be excessive to use those same data to calculate the increase (over PN_t) in Year (t+1) to arrive at $P_{(t+1)}$. The best way to address this issue is to simply apply the Commission's Year (t+1) limitation to P_t rather than to PN_t:

Annual Limitation for Postal Price in Year (t+1) = CPI_t / $CPI_{(t-1)}$ – 1, and

$P_{t+1} = P_t * (CPI_t / CPI_{(t-1)})$

Where:

CPI_t = Actual average monthly CPI-U for Year (t)
$CPI_{(t-1)}$ = Actual average monthly CPI-U for Year (t-1).

II. Calculation Of Year-to-Year Percentage CPI-U Changes

(a) Overview

Subsection 3100.21 explains the Commission's proposed procedure for calculating the annual average price limitation: generate the annual average of the monthly CPI-Us for each of the two previous years and then determine the year-over-year percent change in the two averages. This result is equivalent to a weighted average of the year-to-year CPI-U percent increases for each month, where the weights are based on the CPI-U values for the rate year before the one just completed (if the rate year is year t, then the weights come from year t-2). o By contrast, the correct approach would be to determine the percent increase for each of the 12 preceding months and then to simply average all 12 percentages (multiply each value by 1/12).

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so this asymmetry should not be a big concern. However, we simply note that the Commission has no rules in place to make mid-year corrections if the inflation rate declines over an extended period of time.

The correct, unweighted average approach will yield an unbiased estimate of the true annual inflation rate while the weighted average will not generally.[8] This is because the CPI-U weights as well as the month-to-SPLY (same period last year) percent increases in the CPI-U are both subject to variation. However, if the weights applied are constant (such as with the correct approach which weights each month's percentage increase the same), the CPI-U percent change calculation will always be an unbiased estimate of the true (or expected) inflation rate as long as the latter remains constant. Further, the monthly CPI-U weights in the Commission's approach could lead to greater instability (higher variances) in the price cap (percent inflation change) calculations compared to the simple average. This is because both the CPI-U weights and the month-to-SPLY percentage changes would be subject to variation, whereas with the simple average, only the latter would be subject to such variation.[9]

(b) Statistical Superiority Of Using Simple Average Percentage

The Commission's proposed moving average method for calculating annual percentage price increases at the start of Rate Year (t) implicitly involves weighting the individual month-to-SPLY (same period last year) annual percentage increases by the CPI values in Year (t-2), as follows:

$$CPI_{(t-1)}/CPI_{(t-2)} = (CPI_{(t-1)1}/CPI_{(t-2)1}) * w_{(t-1)1} + (CPI_{(t-1)2}/CPI_{(t-2)2}) * w_{(t-1)2} + \ldots$$

$$+ (CPI_{(t-1)12}/CPI_{(t-2)12}) * w_{(t-1)12}, \qquad (1)$$

where:

$w_{(t-1)1} = CPI_{(t-2)1}/(CPI_{(t-2)1} + CPI_{(t-2)2} + \ldots + CPI_{(t-2)12})$,
$w_{(t-1)2} = CPI_{(t-2)2}/(CPI_{(t-2)1} + CPI_{(t-2)2} + \ldots + CPI_{(t-2)12})$, ... ,
$w_{(t-1)12} = CPI_{(t-2)12}/(CPI_{(t-2)1} + CPI_{(t-2)2} + \ldots + CPI_{(t-2)12})$.

However, any estimate should reflect the expected value of the annual inflation rate for Year (t). Unfortunately, it is not clear that the Commission's moving average method achieves this desired result, because of variation in both month-to-SPLY percentage price increases and the corresponding weights.

However, any bias can be eliminated by assigning fixed values to weights. Therefore a simple moving average method for calculating percentage price increases for Year (t) is proposed to replace the Commission's approach. Under this approach,

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[8] The true annual inflation rate is the one that can be expected over time with any seasonal effects netted out. In particular, the actual annual inflation rate at any time t will be $i_t = r + e_t$ where e_t is an error term and r is the true inflation rate. Taking expected values: $E(i_t) = E(r) + E(e_t)$ or $E(i_t) = r$, a constant, since $E(e_t) = 0$. So the expected value of the actual inflation rate is the true rate r.

[9] In the comments on the partial-year limitation, we used the weighted-average approach to determining percentage CPI-U change. They can also be adapted to use the simple-average approach.

the percentage price increase for each month is assigned a fixed value of 1/12. This will result in unbiased inflation estimates as explained below.

First, let the actual inflation rate for some month (i) in year (t) be measured by $CPI_{ti}/CPI_{(t-1)i} - 1 = r_{ti}$, where $r_{ti} = r + e_{ti}$. This statement explains the actual annual inflation rate at any month (i) in the current year from the same point in time in the previous year as equal to a constant (r) plus an error term e_{ti} where the expected value of the error term is zero, i.e. $E(e_{ti}) = 0$.[5] Then rearranging to $CPI_{ti}/CPI_{(t-1)i} = 1 + r + e_{ti}$, shows the expected value of any month-to-SPLY adjustment factor as:

$$E(CPI_{ti}/CPI_{(t-1)i}) = E(1 + r) + E(e_{ti})$$

$$= 1 + r,$$

from $E(e_{ti}) = 0$. But since $E(r_{ti}) = r$ by the same derivation, then:

$$E(CPI_{ti}/CPI_{(t-1)i}) = 1 + E(r_{ti}).$$

This shows also that the expected value of any month-to-SPLY adjustment factor is one plus the expected value of the inflation rate, a constant (r). Similarly, $CPI_{(t-1)i}/CPI_{(t-2)i} - 1 = r_{(t-1)i}$, where $r_{(t-1)i} = r + e_{(t-1)i}$. So by the same procedure:

$$E(CPI_{(t-1)i}/CPI_{(t-2)i}) = 1 + r$$

$$= 1 + E(r_{(t-1)i}),$$

and, therefore, $r = E(r_{ti}) = E(r_{(t-1)i})$. This last statement says that the expected values for the annual inflation rates measured at different points in time (t or t-1) and in different months (i values can be different across the two years) are all equal to the same constant (r).

Now, suppose at the beginning of rate year (t), that an estimate of the annual inflation rate for that year r_{test} is formed according to the following simple average of the month-to-SPLY inflation rates for the previous year:

$$r_{test} = (CPI_{(t-1)1}/CPI_{(t-2)1}) / 12 + (CPI_{(t-1)2}/CPI_{(t-2)2}) / 12 + ... + (CPI_{(t-1)12}/CPI_{(t-2)12}) / 12 - 1.$$

Then taking the expected value of both sides:

$$E(r_{test}) = E[(CPI_{(t-1)1}/CPI_{(t-2)1}) / 12] + E[(CPI_{(t-1)2}/CPI_{(t-2)2}) / 12] + ...$$

$$+ E[(CPI_{(t-1)12}/CPI_{(t-2)12}) / 12] - E[1]$$

[5] A zero expected value for the error term is a standard statistical assumption used in deriving expected values for non-zero random values of other terms.

$$= E[(CPI_{(t-1)1}/CPI_{(t-2)1})] / 12 + E[(CPI_{(t-1)2}/CPI_{(t-2)2})] / 12 + \ldots$$

$$+ E[(CPI_{(t-1)12}/CPI_{(t-2)12})] / 12 - 1$$

$$= r,$$

using $E[(CPI_{(t-1)i}/CPI_{(t-2)i})] - 1 = r$ for all terms shown. But since $r = E(r_{ti}) = E(r_{(t-1)i})$ also, then:

$$E(r_{test}) = E(r_{ti}),$$

for all i from 1 to 12. The result shows that the expected value for the inflation rate estimate in year (t) formed using the simple moving average method (the left hand side of the expression) is the unvarying expected value of the annual inflation rate that can be formed at any month in the rate year, all equal to the constant (r). This proves that the calculation generates an unbiased estimate of the expected value of that rate.

By contrast, the difficulty with the Commission's proposed weighted moving average method can be shown by first expressing (1) as:

$$CPI_{(t-1)}/CPI_{(t-2)} - 1 = (CPI_{(t-1)1}/CPI_{(t-2)1} - 1) * w_{(t-1)1} + \ldots + (CPI_{(t-1)12}/CPI_{(t-2)12} - 1) * w_{(t-1)12}.$$

Then, as above, taking expected values:

$$E[CPI_{(t-1)}/CPI_{(t-2)} - 1] = E[(CPI_{(t-1)1}/CPI_{(t-2)1} - 1) * w_{(t-1)1}] + \ldots$$

$$+ E[(CPI_{(t-1)12}/CPI_{(t-2)12} - 1) * w_{(t-1)12}]$$

$$= E[r_{(t-1)1} * w_{(t-1)1}] + E[r_{(t-1)2} * w_{(t-1)2}] + \ldots$$

$$+ E[r_{(t-1)12} * w_{(t-1)12}]. \qquad (2)$$

The problem with the Commission's approach now becomes more evident because the individual month weights and the corresponding month-to-SPLY inflation rates both vary and these variations are not necessarily independent. If these variations were independent, then the last expression would indeed reduce to an unbiased estimate of the expected annual inflation rate. However, this generally will not be the case.

To be clearer about the difficulty, consider for any month (i) the multiplicative term $r_{(t-1)i} * w_{(t-1)i}$. Since $CPI_{(t-2)i}$ appears in both $r_{(t-1)i}$ and $w_{(t-1)i}$, any variation in $CPI_{(t-2)i}$ alone caused by variations in $r_{(t-2)i}$ will cause both terms to vary non-independently. The degree of non-independent variation in turn affects the expected value for the product of the term. In particular, the covariance between $r_{(t-1)i}$ and $w_{(t-1)i}$ can be used to define the degree of non-independent variation and the resulting effect on the expected value of $r_{(t-1)i} * w_{(t-1)i}$ according to:

$$E[r_{(t-1)i} * w_{(t-1)i}] = COV[r_{(t-1)i}, w_{(t-1)i}] + E[r_{(t-1)i}] * E[w_{(t-1)i}]$$

$$= COV[r_{(t-1)i}, w_{(t-1)i}] + r * E[w_{(t-1)i}].$$ [10]

Substituting into (2) then gives:

$$E[CPI_{(t-1)}/CPI_{(t-2)} - 1] = \varphi COV[r_{(t-1)i}, w_{(t-1)i}] + r * \varphi E[w_{(t-1)i}]$$
$$= \varphi COV[r_{(t-1)i}, w_{(t-1)i}] + r,$$

where the covariances are summed across all (i) from 1 to 12. It therefore becomes apparent that an unbiased estimate for the weighted moving average approach requires $\varphi COV[r_{(t-1)i}, w_{(t-1)i}] = 0$. This in general will not hold because of $COV[r_{(t-1)i}, w_{(t-1)i}] \neq 0$ for for each (i) for the reasons explained above.

Respectfully submitted,

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[10] The link between the expected value of a product of two random terms and the product of their expected values can be explained as follows. The covariance of any two random variables X, Y is defined by:

$$COV(X, Y) = E[(X – E(X))*(Y - E(Y))].$$

Muliply through the right hand side to get:

$$COV(X, Y) = E[X*Y – X*E(Y) – E(X)*Y + E(X)*E(Y)].$$

Taking expected values of each of the terms on the right then gives:

$$COV(X, Y) = E(X*Y) – E(X)*E(Y) – E(X)*E(Y) + E(X)*E(Y) = E(X*Y) – E(X)*E(Y).$$

Then rearrangement and substituting $r_{(t-1)I}$ and $w_{(t-1)i}$ for X and Y gives the term indicated above.