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ON THE NONLINEAR PRICING OF INPUTS*

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1. INTRODUCTION

There has recently been a substantial amount of research devoted to the study of nonlinear pricing in general and two-part tariffs in particular. Leland and Meyer [1976] have demonstrated under fairly general conditions[2] that a two-part tariff is welfare superior to a uniform price. Spence [1977] began the study of general nonlinear outlay schedules. Willig [1978] has demonstrated that there exists such a schedule which is *Pareto* superior to a uniform price not equal to marginal cost. Goldman, Leland, and Sibley [1977] and Roberts [1979] have established the result that a welfare optimal general nonlinear outlay schedule must present the largest buyer with a marginal price equal to marginal cost. This result, which has as its counterpart in the optimal taxation literature the requirement that the marginal tax rate be zero for the highest income bracket (see Cooter [1978], for example), is also a corollary of Willig's result.

However, all of these results have been established using models which posit no economic interactions between purchasers of the nonlinearly priced commodities. In this paper we relax this (implicit) assumption by postulating a model in which the good in question is produced by an upstream monopolist and purchased as an input by firms which sell their output in a perfectly competitive downstream market. This generates indirect interactions between purchasers since the marginal price faced by *any* firm affects the equilibrium output price and hence the input demand of *all* firms. These indirect effects have important implications for the optimal pricing strategy of the upstream firm.

In Section 2 we recast the welfare analysis of the simple two-part tariff using the classical model of perfect competition in which all firms are identical and free entry and exit ensures that the equilibrium output price is equal to minimum average cost. In this context we discover that two-part tariffs are *not* generally desirable from a welfare standpoint, as the Leland and Meyer analysis would

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[2] Theirs and all the other results referred to in the text depend on the assumption that buyers cannot resell the commodity, for otherwise implementation of the nonlinear pricing schedule would be frustrated by arbitrage.

suggest.[3] Rather, their desirability depends crucially on the properties of the underlying production technology. This is due to the fact that the entry fee, instead of acting as a "lump sum levy," affects both the equilibrium number of firms and their output level. This new distortion must be balanced against the losses due to a unit price in excess of marginal cost. We provide readily interpretable sets of necessary and sufficient conditions for two-part tariffs to be welfare superior to a uniform price. Most surprisingly, for the empirically relevant class of production processes in which the purchased input is required in fixed proportion to output, we discover that a two-part tariff is *never* optimal from either a profit or welfare maximizing standpoint.

In Section 3 we extend our analysis to encompass general nonlinear outlay schedules by introducing firm heterogeneity in the context of a perfectly competitive industry with Ricardian rents. Our principal result here[4] is that the profit or welfare maximizing outlay schedule requires that marginal price be *everywhere* greater than marginal cost. Thus the efficiency result cited earlier, i.e., marginal price equals marginal cost for the largest user, does not hold in our interactive formulation. Intuitively, this new result follows from the fact that, here, discounts offered on the last units sold to the largest purchaser *do* impact the revenues that the monopolist receives from its other customers. For the discount prompts the large firm to expand its supply, thereby reducing the equilibrium output price and, hence, the purchases of all the smaller firms.

2. TWO-PART TARIFFS FOR INPUTS: THE CASE OF IDENTICAL FIRMS

The classic case of the perfectly competitive industry characterized by the free entry and exit of identical firms operating at the minimum point of a U-shaped average cost curve provides an ideal starting point for our analysis. Since all firms are identical, the only variables directly affected by pricing policy are the number of firms purchasing the input and the quantity they select. Thus the simple two-part tariff, consisting of an entry fee e and a unit price r, is also the most general nonlinear outlay schedule which the upstream monopolist need consider.

2.1. *The Basic Model.* There are many equivalent ways of characterizing downstream industry equilibrium for competitive firms with access to a freely available technology facing a two-part tariff for one of its inputs. For our purposes it proves most convenient to employ the McFadden-Lau profit function $\tilde{\pi}(p, r, w)$, where p is the (endogenously determined) price of the final product and w is the vector (henceforth suppressed) of prices for other factors of production. To ensure against triviality, we assume that the input in question is *essential.*

[3] This phenomenon was recently noted by Schmalensee [1981] in a similar context.

[4] In an earlier note we constructed a similar but less general model in which Willig's [1978] Pareto-superiority result does not obtain; see Ordover and Panzar [1980].

No positive level of output can be produced without it. Equilibrium in the final product market is then determined by

$$\pi(p, r, e) \equiv \tilde{\pi}(p, r) - e = 0 \tag{1}$$

and

$$ny(p, r) - D(p) = 0, \tag{2}$$

where n is the equilibrium number of firms in the industry, $D(p)$ is the demand curve for the final product, and $y(p, r)$ is the *firm* supply function.

At this point it is also worthwhile to recall some of the basic properties of $\tilde{\pi}$ and, hence, π:

$$\frac{\partial \pi}{\partial p} = \frac{\partial \tilde{\pi}}{\partial p} = y(p, r) \tag{3}$$

$$\frac{\partial \pi}{\partial r} = \frac{\partial \tilde{\pi}}{\partial r} = -x(p, r) \tag{4}$$

$$\frac{\partial^2 \pi}{\partial r \partial p} = -x_p = y_r = \frac{\partial^2 \pi}{\partial p \partial r} \tag{5}$$

where x is the firm's demand function for the input supplied by the monopolist. It will prove convenient to establish some notation for the structural properties of this characterization of the productive technology in terms of elasticities:

$$\delta \equiv \frac{py_p}{y} > 0 \tag{6}$$

$$\eta \equiv -\frac{rx_r}{x} > 0 \tag{7}$$

$$\alpha \equiv \frac{px_p}{x}. \tag{8}$$

The elasticity of input demand with respect to *output* price α will play an important role in our analysis. Its qualitative sign is indeterminate a priori, but is readily interpreted in terms of the traditional expansion path of the firm. It can easily be shown that

$$\alpha = \eta_y \delta \tag{9}$$

where η_y is the elasticity with respect to output of the Samuelsonian constant-output input demand function $\bar{x}(y, r)$. By standard classification, the input x is said to be *inferior* if $\eta_y < 0$, *normal* if $0 \leq \eta_y \leq 1$, and *superior* if $\eta_y > 1$. Using (9), these definitional boundaries are, respectively, given by $\alpha < 0$, $0 \leq \alpha \leq \delta$, and $\alpha > \delta$. Finally, $\varepsilon \equiv -pD'/D > 0$ denotes the elasticity of demand for the *final* product.

With this notation in hand we are ready to perform some comparative statics analysis in order to describe the response of the equilibrium values of p and n to changes in the parameters r and e. Using (1), we have immediately,

$$\frac{\partial p}{\partial e} = \frac{1}{y} > 0 \tag{10}$$

and

$$\frac{\partial p}{\partial r} = \frac{x}{y} > 0, \tag{11}$$

which amount to a slight extension to the case of a fixed charge, of the usual result (see, e.g., Silberberg [1974]) that the equilibrium output price is an increasing function of the price of an input.

The effects on the equilibrium number of firms are somewhat more complicated. Totally differentiating (2) and substituting (10) and (11) yields

$$\frac{\partial n}{\partial e} = -\frac{n}{py}(\varepsilon + \delta) < 0 \tag{12}$$

and

$$\frac{\partial n}{\partial r} = \frac{nx}{py}(\alpha - \delta - \varepsilon). \tag{13}$$

An increase in the fixed charge affects the *average* but not the *marginal* cost curve of the firm, and results in increased optimal firm output. But, because $\partial p/\partial e > 0$, the market demand for the final product declines. This smaller demand will be produced by a smaller number of firms each producing a larger output.

An increase in the input price has an ambiguous affect on the equilibrium number of firms because two effects are at work. Total market demand falls, since $\partial p/\partial r > 0$. If the input is inferior or normal, then the equilibrium firm size rises, and both forces, operating in the same direction, ensure that $\partial n/\partial r < 0$. If the input is superior ($\alpha > \delta$), however, the optimal output per firm shrinks, and the number of firms may increase provided that the final demand is not too elastic. If, in percentage terms, optimal firm size shrinks by more than market demand, the equilibrium number of firms must rise. These are the forces at work in (13).

We conclude this discussion of comparative statics results by demonstrating a symmetry property which will be important in subsequent analysis.

LEMMA 1. *The change in industry demand, $X \equiv nx$, resulting from an increase in the entry fee is precisely equal to the change in the number of firms due to an increase in the unit price; i.e., $\partial X/\partial e = \partial n/\partial r$.*

PROOF.
$$\frac{\partial X}{\partial e} = n\left(x_p \frac{\partial p}{\partial e}\right) + x\frac{\partial n}{\partial e} = \frac{nx_p}{y} + x\frac{\partial n}{\partial e}.$$

Using (12), this becomes

$$\frac{\partial X}{\partial e} = -\frac{nx}{py}(\varepsilon + \delta) + \left(\frac{nx}{py}\right)\frac{px_p}{x} = \frac{nx}{py}(\alpha - \delta - \varepsilon) = \frac{\partial n}{\partial r}. \tag{14}$$

Q. E. D.

2.2. *Welfare Analysis.* We wish to study optimal choices of input two-part tariffs under a variety of objective functions. Since, by assumption, downstream firms always earn zero profits, we take as our welfare measure a weighted sum of upstream monopoly profits and the surplus of final consumers:

$$W = \gamma \int_p D(p')dp' + (1-\gamma)[(r-m)X + en - F] \tag{15}$$

where m and F are, respectively, the (constant) marginal and fixed costs of the input monopolist and $0 \leq \gamma < 1/2$. This formulation allows us to encompass both the analysis of monopoly profit maximization ($\gamma = 0$) and Ramsey-type constrained welfare optima ($0 < \gamma < 1/2$). The case of unconstrained surplus maximization, $\gamma = 1/2$, is uninteresting in this context. Its solution would yield $e = 0$, $r = m$ and an upstream deficit of F.

Necessary conditions for an optimum are given by

$$\frac{\partial W}{\partial e} = -\gamma D(p)\frac{\partial p}{\partial e} + (1-\gamma)\left[(r-m)\frac{\partial X}{\partial e} + n + e\frac{\partial n}{\partial e}\right] \leq 0; \tag{16}$$

$$e \geq 0,\ e\frac{\partial W}{\partial e} = 0.$$

$$\frac{\partial W}{\partial r} = -\gamma D(p)\frac{\partial p}{\partial r} + (1-\gamma)\left[(r-m)\frac{\partial X}{\partial r} + X + e\frac{\partial n}{\partial r}\right] \leq 0; \tag{17}$$

$$r \geq 0,\ r\frac{\partial W}{\partial r} = 0.$$

Using (2), (10), and (11), these immediately simplify to

$$\frac{\partial W}{\partial e} = (1-2\gamma)n + (1-\gamma)(r-m)\frac{\partial X}{\partial e} + (1-\gamma)e\frac{\partial n}{\partial e} \tag{18}$$

$$\frac{\partial W}{\partial r} = (1-2\gamma)X + (1-\gamma)(r-m)\frac{\partial X}{\partial r} + (1-\gamma)e\frac{\partial n}{\partial r}. \tag{19}$$

The first contrast between two-part tariffs for inputs and those for final products is highlighted by the simple structure of our model. Were the market in question a final product one, the assumptions of identical consumers and essentiality would suffice for the well-known Coasian result in which the unit price is set equal to marginal cost and profits are extracted via the entry fee. In our model, however, the input is essential but *firms* are not, as revealed by our comparative statics analysis. So that, not surprisingly, this simple result rarely pertains.

PROPOSITION 1. *A "perfect" two-part tariff (i.e., $e > 0$, $r = m$) can be optimal only if the purchasing firms are operating in a region where input demand is unresponsive to the level of output and output price; that is, $\alpha = \eta_y = 0$.*

PROOF. Assume, arguendo, that $e > 0$, $r = m$ are optimal. Then from (18)–(19), we must have

$$\frac{\partial W}{\partial e} = (1-2\gamma)n + (1-\gamma)e\frac{\partial n}{\partial e} = 0 \tag{20}$$

and

$$\frac{\partial W}{\partial r} = (1-2\gamma)nx + (1-\gamma)e\frac{\partial n}{\partial r} = 0. \tag{21}$$

Substituting (20) into (21) yields the requirement

$$\frac{\partial W}{\partial r} = (1-\gamma)e\left(\frac{\partial n}{\partial r} - x\frac{\partial n}{\partial e}\right) = 0. \tag{22}$$

Using (12) and (13), this becomes

$$\frac{\partial W}{\partial r} = (1-\gamma)e\left(\frac{nx}{py}\right)\alpha = 0. \tag{23}$$

Q. E. D.

We turn now to examine whether or not *any* two-part tariff is preferable to a uniform price. Leland and Meyer [1976] found that, with no income effects, it always paid a final product monopolist to introduce a positive entry fee. Schmalensee [1981] later extended this result to the case of an objective function of the form employed here. However, he also argued that this need not be the case for an input monopolist. We now present a precise, readily interpretable condition for a two-part tariff to dominate a uniform price.

PROPOSITION 2. *If the total derivative of input usage with respect to input price is negative* ($dx/dr = x_p \partial p/\partial r + x_r < 0$), *then a uniform price cannot be optimal.*

PROOF. Suppose $e=0$, $r>0$. Then from (19), we must have

$$\left.\frac{\partial W}{\partial r}\right|_{e=0} = (1-2\gamma)X + (1-\gamma)(r-m)\frac{\partial X}{\partial r} = 0. \tag{24}$$

Multiplying (18) by x yields

$$x\left.\frac{\partial W}{\partial e}\right|_{e=0} = (1-2\gamma)X + (1-\gamma)(r-m)x\frac{\partial X}{\partial e}. \tag{25}$$

Substituting from (24) yields

$$x\left.\frac{\partial W}{\partial e}\right|_{e=0} = (1-\gamma)(r-m)\left(x\frac{\partial X}{\partial e} - \frac{\partial X}{\partial r}\right)$$

$$= (1-\gamma)(r-m)\left(x\frac{\partial X}{\partial e} - x\frac{\partial n}{\partial r} - n\frac{dx}{dr}\right).$$

$$x\left.\frac{\partial W}{\partial e}\right|_{e=0} = -(1-\gamma)(r-m)n\frac{dx}{dr} \tag{26}$$

from Lemma 1. Thus the necessary condition for an optimum at $e=0$, $\partial W/\partial e \leq 0$, cannot be satisfied if $dx/dr<0$. Q. E. D.

The fact that dx/dr is not always negative and a two-part tariff generally desirable in this model may be surprising at first. After all, don't all input demand functions exhibit negative own-price derivatives? The difference here is that the derivative in question is total rather than partial in that it includes the indirect effect on input demand via the equilibrium *output* price response to an input price change. As Silberberg [1974] has pointed out, *total* own-price effects need not be negative when the equalizing role of the output price is recognized. To see this, we note that

$$\frac{dx}{dr} = x_p \frac{\partial p}{\partial r} + x_r = \frac{x}{y} x_p + x_r. \tag{27}$$

While $x_r<0$, x_p may be of either sign. Thus we have immediately that a two part-tariff is required for optimality if the input is inferior ($x_p<0$). Somewhat surprisingly, this result can be extended.

PROPOSITION 3. *A uniform price can be optimal* only if *the input is strictly normal over the relevant range. That is* $0<\alpha\leq\delta$, *or equivalently* $0<\eta_y\leq 1$.

PROOF. Inspection of (27) establishes the result for the case $\alpha \equiv px_p/x \leq 0$. For superior inputs, we exploit the convexity of $\tilde{\pi}(p, r)$:

$$\tilde{\pi}_{rr}\tilde{\pi}_{pp} - \tilde{\pi}_{rp}^2 = -x_r y_p - x_p^2 \geq 0, \tag{28}$$

which yields

$$x_r \leq -\frac{x_p^2}{y_p}. \tag{29}$$

Upon substituting this into (27), we obtain

$$\frac{dx}{dr} \leq x_p\left(\frac{x}{y} - \frac{x_p}{y_p}\right) = \frac{xx_p}{py_p}(\delta-\alpha). \tag{30}$$

The r.h.s. of (30) is negative only if $\alpha<0$ or $\alpha>\delta$. Q. E. D.

We have left the case in which the technology requires that x and y vary in fixed proportion, i.e., $\delta=\alpha$ or $\eta_y=1$, for special discussion since the nature of the optimal two-part tariff there depends crucially upon the smoothness properties of the underlying technology with respect to x:

PROPOSITION 4. *If the underlying production function is strongly quasi-concave and homothetic, there exists a two-part tariff superior to a uniform price.*

PROOF. Under these hypotheses Silberberg [1974] has shown that $dx/dr<0$, in which case Proposition 2 applies. Q.E.D.

For the interesting and empirically relevant case in which the input x (but not *all* others) is required in fixed proportions for the production of y, we obtain precisely the opposite result!

PROPOSITION 5. *Given fixed proportions between x and y; i.e., $C(y, r, w)= rzy+\psi(y, w)$; the optimal entry fee must be zero.*

PROOF. Under these conditions, $y(p, r, w)=f(p-rz, w)$. Letting $f'>0$ represent the derivative of f with respect to its first argument, we have

$$\frac{\partial y}{\partial p}=f'; \quad \frac{\partial y}{\partial r}=-zf'=-z\frac{\partial y}{\partial p}. \tag{31}$$

Substituting (31) and the identity $zy=x$ into (10) and (11) yields

$$\frac{\partial p}{\partial r}=z; \quad \frac{\partial p}{\partial e}=\frac{1}{y}. \tag{32}$$

Totally differentiating (2) and solving using (31) and (32), we obtain

$$\frac{\partial n}{\partial r}=\frac{z}{y}D', \quad \frac{\partial n}{\partial e}=\frac{D'-nf'}{y^2}. \tag{33}$$

Inserting these results into (18) and (19), using Lemma 1 leaves us with

$$\frac{\partial W}{\partial e}=(1-2\gamma)n+(1-\gamma)\left[\frac{(r-m)zD'}{y}+\frac{e(D'-nf')}{y^2}\right] \tag{34}$$

and

$$\frac{\partial W}{\partial r}=(1-2\gamma)nzy+(1-\gamma)\left[(r-m)z^2D'+\frac{ezD'}{y}\right]. \tag{35}$$

Substituting $(\partial W/\partial r)/zy=0$ into (34) yields

$$\frac{\partial W}{\partial e}=-(1-\gamma)\frac{enf'}{y^2}\leq 0. \tag{36}$$

Since, with the exception of e, all terms on the r.h.s. of (36) are strictly positive, the only way the necessary condition $e(\partial W/\partial e)=0$ can be satisfied is with $e=0$.

Q.E.D.

The intuition behind this result is rather straightforward. It is well-known that under fixed proportions an upstream uniform pricing input monopolist can extract all the profits which an integrated uniform pricing monopolist could reap. Since competition downstream ensures that a uniform price prevails in the final product market, there can be nothing to gain from introducing a two-part tariff;

optimal choice of r allows the monopolist to earn the maximum possible under such circumstances. There is something to lose, however, since an entry fee $e>0$ causes the downstream firms to operate at an inefficiently large scale. Total (upstream plus downstream) costs are not minimized and a portion of this deadweight burden falls on the monopolist. Viewed another way, this result reveals the futility of attempting to impose a seemingly nondistortionary lump-sum levy e on a perfectly competitive industry with free entry and exit. See Carlton and Loury [1980] for another example of this principle.

The qualitative results thus far are valid for both profit maximizing monopolists and welfare maximizing firms bound by profit requirements. Thus there is a clear implication that pricing rules for profit and (constrained) welfare maximizing monopolists in some sense "look" the same. The policy issue which our analysis has yet to address concerns the desirability, as measured by total surplus, of allowing a uniform pricing profit maximizing monopolist to introduce a two-part tariff. Absent strong regularity assumptions on the underlying structural model it is impossible, in general, to compare welfare levels generated by a profit maximizing monopolist with and without the ability to offer two-part tariffs. (See Leland and Meyer [1976] for some simulation results on this subject). However, our analysis does allow us to deduce something about the relative *marginal* social and private incentives to introduce two-part pricing.

PROPOSITION 6. *With respect to an initial uniform pricing equilibrium with $r>m$, the private* marginal *incentive to introduce a two-part tariff always exceeds the social one.*

PROOF. In our formulation, W equals one-half total consumers' plus producer's surplus, π^m+S when $\gamma=1/2$. Therefore, using (19) we have

$$\left.\frac{\partial(\pi^m+S)}{\partial e}\right|_{e=0} = 2\left.\frac{\partial W}{\partial e}\right|_{\substack{\gamma=\frac{1}{2}\\ e=0}} = (r-m)\frac{\partial X}{\partial e}. \tag{37}$$

Whereas

$$\left.\frac{\partial\pi^m}{\partial e}\right|_{e=0} = n + (r-m)\frac{\partial X}{\partial e} > \frac{\partial(\pi^m+S)}{\partial e}. \tag{38}$$

Q.E.D.

Thus it seems safe to conclude that there may be cases in which it pays a profit-maximizing monopolist to introduce a two-part tariff which lowers total surplus.

3. OPTIMAL OUTLAY SCHEDULE FOR AN INPUT: THE CASE OF HETEROGENEOUS FIRMS

In Section 2 we analyzed the welfare implications of supplanting a uniform price scheme with a two-part tariff. We noted that when all buyers have identical cost functions, more precisely, identical derived demand functions for the input, a two-part tariff is the only relevant alternative to a uniform price. In this section we relax the homogeneity assumption and postulate instead that firms differ in their cost functions. Once heterogeneity among the input buyers is admitted, the set of possible pricing schemes encompasses not only a uniform price and two-part tariffs but also other more complex arrangements. In fact, a two-part tariff is usually not the best price schedule that can be implemented. In this section, we allow the purveyor of the input — the upstream monopolist — to choose any pricing schedule, subject, however, to various constraints which we shall spell out below. The resulting price schedule is referred to as the (optimal) outlay schedule for an input and is denoted by $R(x)$. We show that the properties of $R(x)$ differ in important respects from the properties of the optimal outlay schedules for outputs, (see Willig [1978], Roberts [1979]) or, for that matter, from the optimal income tax schedules, (see Mirrlees [1971], Cooter [1978]).

In particular, we demonstrate that when uniform marginal cost pricing is infeasible, all firms face marginal prices for the input which exceeds its marginal cost. *This finding is in contrast to the usual result found in the optimal pricing* literature. The classic result (see Willig [1978]) states that any Pareto-optimal outlay schedule must have the property that the marginal price paid by the largest purchaser must equal marginal cost. (The optimal income tax analogue is that the marginal income tax is equal to zero at the top of the income tax schedule.) This difference in results arises because in our model a discount offered to the largest buyer leads it to increase its output, depressing the market price and the input purchases of smaller firms. Previous models have not allowed for the possibility of such indirect economic interactions.

We do not wish to imply, however, that quantity discounts to large buyers may be welfare suboptimal. To the contrary: For a one-to-one cost function we are able to demonstrate that the marginal price paid by the largest user of the input, while still above the marginal cost, is lower than the marginal prices paid by all other users of the input. This implies that, at least locally, the optimal outlay schedule is characterized by quantity discounts.

3.1. *Preliminaries.* Before $R(x)$ can be established, the model of the downstream industry must be cast in a form which will permit us to characterize the diversity in individual firm technologies. We do this by indexing the structural, competitive profit function $\tilde{\pi}$ with the cost reducing parameter $t \in [0, T]$. Thus $\tilde{\pi} = \tilde{\pi}(p, r, t)$, with $\partial\tilde{\pi}/\partial t \equiv \tilde{\pi}_t > 0$ implying that greater levels of t make it possible for the firm to achieve higher (maximized) profits, given output and input prices. We assume that t measures the firm's endowment of a productive fixed factor which is normal and complementary with x for all values of r and all levels of output. This implies that favored firms are unambiguously "bigger" in the sense that they supply more output and demand more of the monopolist's input. That is, using the derivative properties of $\tilde{\pi}$,

$$(39) \qquad \frac{\partial^2 \tilde{\pi}}{\partial p \partial t} \equiv \frac{\partial y(p, r, t)}{\partial t} \equiv y_t > 0;$$

$$-\frac{\partial^2 \tilde{\pi}}{\partial r \partial t} \equiv \frac{\partial x(p, r, t)}{\partial t} \equiv x_t > 0.$$

We assume that the least efficient firm is just viable when the input is uniformly priced at marginal cost m; i.e.,

$$(40) \qquad \tilde{\pi}(p^e, m, 0) = 0,$$

where p^e is the resulting equilibrium price in the downstream market.

In order to develop a manageable optimal control formulation, let us assume for a moment that the optimal $R(x)$ schedule is known. When confronted with such a schedule a firm of type t has derived supply and demand functions $\hat{y}(p, t)$ and $\hat{x}(p, t)$ which result from the solution of the program:

$$(41) \qquad \max_{y,x} py - V(y, w, x, t) - R(x)$$

where V is the minimum expenditure on other inputs, the variable cost function representation of the technology. Let us denote by $r(t)$ the slope of the outlay function evaluated at $\hat{x}(p, t)$; i.e. $r(t) \equiv R'[\hat{x}(p, t)]$. Thus, $r(t)$ is the marginal cost of the input to a firm of type t.[5]

Observe that if the monopolist could identify a firm's t it would be able to induce the same supply and demand behavior by the firm. This follows from the fact that, given that the firm is producing, only the marginal properties of the outlay schedule matter. Therefore $\hat{x}(p, t) = x(p, r(t), t) \equiv -\partial\tilde{\pi}/\partial r$ and $\hat{y}(p, t) = y(p, r(t), t) \equiv \partial\tilde{\pi}/\partial p$. Were firm t able to purchase *all* of its x at price $r(t)$, its profits would be given by $\tilde{\pi}(p, r(t), t)$. In general, however, the prices paid for inframarginal units will diverge from $r(t)$ and actual profits $\pi(t)$ will differ from $\tilde{\pi}$; the difference, $\tilde{\pi} - \pi$, which accrues to the monopolist can be viewed as a firm-specific entry fee $e(t)$. From this perspective, then, the outlay schedule $R(x)$ can be viewed as a set of firm specific two-part tariffs, $(e(t), r(t))$: personalized entry fees and marginal prices. When a firm is presented with such a tariff, it purchases the same quantity of input, produces the same output, and earns the same profit as it would if it were optimizing against the impersonal outlay schedule $R(x)$. This observation permits us to convert the problem of choosing the optimal outlay schedule into the problem of choosing the set of optimal firm specific two-part tariffs (see Roberts [1979], for a similar approach). In fact, in what follows it will prove advantageous to treat $\pi(t)$ as being subject to choice, i.e., as a state variable, and let $e(t)$ be implicitly defined as $\tilde{\pi}(p, r(t), t) - \pi(t)$.

[5] Our use of this construction formally limits our analysis to the characterization of the optimal *differentiable* outlay schedule. Given our smoothness assumptions this does not seem overly restrictive. However, proving that the optimal $R(x)$ is differentiable is typically quite difficult and will not be attempted here. See Goldman, Leland, and Sibley [1977].

However, both formulations obscure the important fact that no firm-specific information is available to the monopolist. When the purveyor of the input does not use any such information regarding buyers, the fully decentralized input allocation process guarantees that

$$\dot{\pi} - \frac{\partial \bar{\pi}}{\partial t} = 0, \tag{42}$$

where $\dot{\pi} \equiv d\pi(t)/dt$. Using an argument due to Mirrlees [1971], we can show that this follows from the fact that in a decentralized system a firm of type t' would behave "as if" it were a firm of type t, if by so doing it could earn higher profit. Indeed, for any firm it must be true that the difference between the maximum profit that it can earn $\pi(t)$ and the profit that it earns when it buys the input using its "personalized" two-part tariff is zero. That is,

$$[\bar{\pi}(p, r(t), t) - e(t)] - \pi(t) = 0.$$

If a firm of type t' were to purchase its inputs using the same schedule, it would have to earn no more than its maximum level of profit. That is,

$$[\bar{\pi}(p, r(t), t') - e(t)] - \pi(t') \leq 0.$$

Thus, if we let the index t' range over all the feasible values of t, this shortfall will be minimized when $t' = t$. In other words,

$$t = \operatorname*{argmax}_{t'} [\bar{\pi}(p, r(t), t') - e(t) - \pi(t')]. \tag{43}$$

Equation (42) is necessary for (43) to hold and therefore becomes our "law of motion."

3.2. *The Optimal Control Formulation.* We now have the ingredients needed to characterize the optimal outlay schedule using control theory. In the formulation that follows, $\pi(t)$ is the state variable satisfying the differential equation (42). Our choice variables are the controls $r(t)$, p, and the marginal producing firm type $s \in [0, T)$, $\pi(s)=0$. The market price p is, strictly speaking, not under the direct control of the monopolist, being instead an implicit functional of $r(t)$. However, it is more convenient and instructive to treat p as a decision variable and impose as an additional constraint the condition that demand equal supply in the downstream market.

Our objective function is a weighted sum of downstream profits, consumers' surplus $S(p)$ and the monopolist's profits. These latter are given by

$$\pi^m = \int_s^T \pi^m(t)g(t)dt - F \tag{44}$$

where $g(t)$ is the positive measure of firms of type t and $\pi^m(t)$, the net profit obtained by the monopolist from a firm of type t, is given by

$$\pi^m(t) = \bar{\pi}(p, r(t), t) - \pi(t) + (r(t) - m)x(p, r(t), t). \tag{45}$$

The social welfare function is thus

$$W = \gamma S(p) + \int_s^T [\gamma\pi(t) + (1-\gamma)\pi^m(t)]g(t)dt - F. \tag{46}$$

This formulation reflects the fact that now, unlike in Section 2, the downstream firms earn profits and some welfare weight must be attached to it. To avoid further complications we assign them the same weight γ as we do a dollar of downstream consumers' *surplus. This is a natural assumption* for the case of a perfectly competitive industry.

W is to be maximized via appropriate choices of $r\{t\}$, p, and s subject to the constraint that demand equal supply in the downstream market:

$$D(p) - \int_s^T y(p, r(t), t)g(t)dt \tag{47}$$

and our equation of motion (42). We let $\mu(t)$ denote the costate variable associated with that constraint. Once these constraints are adjoined, our new maximand is

$$L = W + \lambda\left[D(p) - \int_s^T y(p, r(t), t)g(t)dt\right] + \int_s^T \mu(t)(\tilde{\pi}_t - \dot{\pi})dt. \tag{48}$$

To put (48) in a more familiar form, we define the Hamiltonian function

$$\begin{aligned} H(t) &= [\gamma\pi(t) + (1-\gamma)\pi^m(t) - \lambda y(p, r(t), t)]g(t) + \mu(t)\tilde{\pi}_t \\ &= [(2\gamma - 1)\pi + (1-\gamma)(\tilde{\pi} + (r(t) - m)x) - \lambda y]g(t) + \mu(t)\tilde{\pi}_t. \end{aligned} \tag{49}$$

Using (45), (46), (49) and integrating by parts the last integral in (48), we obtain

$$L = \gamma S(p) + \lambda D(p) + \int_s^T [H(t) + \dot{\mu}\pi(t)]dt - [\mu(T)\pi(T) - \mu(s)\pi(s)], \tag{50}$$

where $\dot{\mu} = d\mu(t)/dt$.

Applying the Maximum Principle, we obtain as necessary conditions for optimality the constraints (42) and (47) and[6]

$$\frac{\partial H}{\partial r} = [(1-\gamma)(r(t) - m)x_r - \lambda y_r]g(t) - \mu(t)x_r = 0 \qquad t \in [s, T] \tag{51}$$

$$\dot{\mu}(t) = -\frac{\partial H}{\partial \pi} = (1 - 2\gamma)g(t) \qquad t \in [s, T] \tag{52}$$

$$\frac{\partial L}{\partial p} = -D(p) + \lambda D' + \int_s^T \frac{\partial H}{\partial p}dt \le 0; \; p \ge 0; \qquad p\frac{\partial L}{\partial p} = 0 \tag{53}$$

$$\frac{\partial L}{\partial p} = (1 - 2\gamma)D(p) + \lambda\left(D' - \int_s^T y_p g dt\right) + \int_s^T \{(1-\gamma)(r(t) - m)x_p g + \mu y_t\}dt$$

[6] To avoid unnecessary clutter, we will suppress the arguments of functions where no confusion will result.

plus the transversality conditions

$$\mu(T) = 0 \tag{54}$$

$$H(s) = \{(1 - \gamma)[\tilde{\pi} + (r(s) - m)x] - \lambda y\}g(s) + \mu(s)\tilde{\pi}_s = 0. \tag{55}$$

3.3. *Optimality Results.* The properties of optimal nonlinear outlay schedules for inputs are readily derived via straightforward manipulation of necessary conditions (51)–(55). However, before we can proceed, we require the following

LEMMA 2. *For an optimal schedule, $\lambda \geq 0$ with $\lambda > 0$ unless $\gamma = 1/2$.*

This result is quite intuitive but its proof is surprisingly complicated and is relegated to the Appendix. Here, we attempt only to argue for its plausibility. Nonnegativity would follow by construction if (47) were an *inequality* constraint. Indeed, it is tempting to interpret it that way since an increase in p serves to shift *outward* the willingness to pay functions of all the downstream firms. If the monopolist were not constrained to "quote" a price such that demand was *at least* equal to supply, perceived downstream "sales" (and upstream profits) could be made to grow without bound. Hence one would expect the constraint to be strictly binding except for the limiting case of pure surplus maximization.

This brings us to a familiar result for the case of unconstrained surplus maximization.

PROPOSITION 7. *If $\gamma = 1/2$ the input is priced* uniformly *at marginal cost, there is no entry fee and no downstream firm is excluded from the input market. That is, $r(t) = m$ and $s = 0$.*

PROOF. From (52), $\dot{\mu} = 0$ when $\gamma = 1/2$. Thus, using (54), $\mu(t) = \mu(T) = 0$ and (51) yields

$$r(t) = m + \lambda y_t/(1 - \gamma)x_t = m \tag{56}$$

using Lemma 2. To see that no firm is excluded, note that these arguments also imply that

$$H(s) = (1 - \gamma)\tilde{\pi}[p^e(s), m, s]g(s),$$

where $p^e(s)$ satisfies (47) for $r(t) = m$. (Clearly, $\partial p^e/\partial s > 0$.) Thus, transversality condition (55) can be satisfied only if $s = 0$, since $\tilde{\pi}[p^e(s), m, s] > \tilde{\pi}[p^e(0), m, 0] = 0$ for positive s. Q.E.D.

The reason for this result is obvious: If the redistribution of surplus between the upstream monopolist and downstream consumers and producers is viewed as a *pure transfer* there can be no reason to introduce any distortions into the price schedule. This means that the input price must be equal to marginal cost for all firms *and* that no firm viable at that price be excluded because of a positive entry fee.

Turning to the more novel features of our analysis, we now examine the properties of optimal outlay schedules for inputs when upstream profits are weighed more heavily than downstream surplus, i.e., $0 \le \gamma < 1/2$. As mentioned earlier, this case covers both pure profit maximization and welfare maximization subject to a binding profit constraint. We begin with our main result.

PROPOSITION 8. *Given $\gamma < 1/2$ and that x is a normal input (i.e., $-y_r = x_p > 0$) marginal price exceeds marginal cost for all firms, including the most favored. That is, $r(t) > m$ $\forall t \in [s, T]$.*

PROOF. Solving (51) yields

$$r(t) - m = \frac{\lambda y_r}{(1-\gamma)x_r} + \frac{\mu(t)x_t}{(1-\gamma)x_r g(t)}. \tag{57}$$

From (52) and (54), we know that $\mu(t) < 0$ for $t < T$, and $\lambda > 0$ by Lemma 2. Thus both terms on the right hand side of (57) are strictly positive. Note that for $t = T$, (57) reduces to

$$r(T) - m = \frac{\lambda v_r}{(1-\gamma)x_r} > 0 \tag{58}$$

because $\mu(T) = 0$; nevertheless, the result still holds. Q.E.D.

Equation (57) clearly distinguishes two causes for the excess of marginal price over marginal cost. The second term on the right hand side is the distortion due to the self-selection constraint. The desire to reduce the marginal distortion to the purchaser of type t is tempered by the realization that any such reduction will *also* reduce the revenues collected from larger buyers. However, for the buyer of type T this effect goes away because there are no bigger purchasers who must be considered. This accounts for the marginal price equals marginal cost result at the "top" of the schedule in earlier models of nonlinear pricing, and the analogous zero marginal tax rate result in the optimal taxation literature.

However, in a model which allows for market-generated interactions among users, equation (58) reveals that the distortion cannot be eliminated. A marginal discount to one user *always* impinges upon the revenue which can be collected from other purchasers. Thus, in our model the first term on the right hand side of (57) is, in essence, independent of user size. It reflects the fact that a slight discount in the marginal price for user t will induce it to supply more to the downstream market. This results in a fall in the price of the final product which causes all firms to reduce their purchases of the input.

The magnitude of this distortion differs over t, if at all, solely because of interfirm variation in the ratio of marginal output supply and input demand responses y_r/x_r. Indeed, we showed in Section 2 that when the technology requires z units of x for each unit of y that $x_r/y_r = z$. Thus, in that empirically relevant case, the distortion can truly be divided up into an irreducible minimum plus a purchaser specific effect:

$$r(t) - m = \frac{\lambda}{(1-\gamma)z} + \frac{\mu(t)x_t}{(1-\gamma)x_r g(t)} > \frac{\lambda}{(1-\gamma)z} = r(T) - m. \tag{59}$$

Inspection of (59) yields:

PROPOSITION 9. *In the case of fixed proportions, the mark-up over marginal cost paid by the best endowed firm, while strictly positive, is less than that paid by any other firm.*

4. CONCLUDING COMMENTS

Because of the title of our paper and the details of our model, the reader may come away with the impression that we obtain results sharply distinct from those of earlier writers primarily because we are studying the pricing of inputs rather than outputs. Nothing could be further from the point! Our results emerge because we are analyzing a model in which the purchasers are inextricably bound together by their participation in the marketplace. This suggests that the classic "zero marginal tax rate" results in the optimal taxation literature are not robust to changes in the standard format in which the individuals being taxed have no economic interactions with each other. Recent work by Ordover [1980] and Stiglitz [1981] seems to confirm this suspicion.

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APPENDIX

PROOF *of Lemma* 2: Rearranging necessary condition (51) and multiplying by x_p yields

$$(1-\gamma)(r(t)-m)gx_p = \frac{\lambda x_p y_r g}{x_r} + \frac{\mu x_p x_t}{x_r}. \tag{A1}$$

Integrating (A1) and substituting the result into (53) yields

$$\frac{\partial L}{\partial p} = (1-2\gamma)D + \lambda\left(D' - \int_s^T A(t)g\,dt\right) + \int_s^T \mu\left[y_t + \frac{x_p x_t}{x_r}\right]dt \tag{A2}$$

where $A(t)=(x_r y_p - y_r x_p)/x_r \geq 0$ from the convexity of $\tilde{\pi}$. Therefore, let $B<0$ denote the term multiplying λ in (A2). For a regular interior maximum of program (41), we note that

$$\frac{dx}{dt} \equiv \frac{\partial \hat{x}}{\partial t} = (V_{yt}V_{xy} - V_{yy}V_{xt})/(V_{yy}V_{xx} + V_{yy}R'' - V_{xy}^2) > 0. \tag{A3}$$

(The numerator is positive because of the normality and complementarity of x and the fixed factor; the denominator is positive due to second-order conditions.)

Thus the optimal schedule must have the property that $dx/dt = x_r\dot{r} + x_t > 0$. Therefore $\dot{r} < -x_t/x_r$ and

$$\text{(A4)} \qquad \frac{dy}{dt} = y_t + y_r\dot{r} = y_t - x_p\dot{r} > \frac{x_p x_t}{x_r} + y_t.$$

Substituting this into (A2) yields

$$\text{(A5)} \qquad \frac{\partial L}{\partial p} \geq (1-2\gamma)D + \lambda B + \int_s^T \mu \frac{dy}{dt}\, dt.$$

Integrating by parts yields

$$\text{(A6)} \qquad \frac{\partial L}{\partial p} \geq \lambda B - \mu(s)y(p, r(s), s) \geq \lambda B$$

with the inequalities strict unless $\mu(s) = 0$. Thus $\partial L/\partial p \leq 0$ requires $\lambda \geq 0$. If $\gamma = 1/2$, $\mu(s) = 0$ and $\lambda = 0$ for $p > 0$. If $\gamma < 1/2$, $\mu(s) < 0$ and we must have $\lambda > 0$ for $\partial L/\partial p \leq 0$. Q. E. D.

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