On the nonexistence of Pareto superior outlay schedules

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Willig demonstrated that in a model in which user demands are independent, a uniform price greater than marginal cost can be Pareto dominated by a nonlinear outlay schedule. However, when users are firms of different sizes which compete in final product markets, their demands must be interrelated. In such cases it may be impossible to achieve any such Pareto improvement.

1. Introduction

■ In an important recent paper Willig (1978) demonstrated that, given a uniform price unequal to marginal cost, a nonlinear outlay schedule can always be constructed which makes *every* economic agent *strictly* better off without the necessity of lump-sum transfers. Earlier literature[1] had focused on maximizing *aggregate*, *scalar* welfare measures such as producers' plus consumers' surplus. Although Willig's analysis was carried out at a high level of generality, his assumption of independent user demands makes it difficult to apply his result to policy issues raised by the nonlinear pricing of inputs to firms producing competing final products. The purchases of such firms are clearly interrelated, since quantity discounts offered to large users will shift outward their final product supply curves, which will, in equilibrium, reduce the market share of their smaller rivals. In this note we develop a simple, yet plausible, model which exhibits such demand interrelationships and also has the property that it is impossible to construct *any* nonlinear tariff which is Pareto superior to an undominated uniform price.

2. The model

■ We assume that the monopolist has two types of consumers, both of which are perfectly competitive firms active in the *same* final product market. "Small" firms employ a freely available technology which requires one unit of the input

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[1] See the references cited by Willig.

sold by the monopolist for each unit of the final product. For notational convenience, we shall use q to measure both quantities. The cost function of these "small" firms is then given by $C^s(q,w,r) = wq + V^s(q,r)$, where w is the monopolist's unit price and r is a vector (henceforth suppressed) denoting the prices of other inputs. The market also contains a *fixed number* $\bar{n}_l$, of larger, more efficient firms which have access to some specialized factor in inelastic supply and thus earn economic rents. The cost function of a representative "large" firm is given by $C^l(q,w) = wq + V^l(q)$. We assume that $V^l(q) < V^s(q)$ and $\partial V^l/\partial q \equiv V^l_q < V^s_q \equiv \partial V^s/\partial q$ for all $q > 0$. (This ensures that in equilibrium a "large" firm actually produces more than a "small" firm.)

It is now possible to characterize equilibrium in the final product market for any uniform price w set by the monopolist. Since the technology used by small firms is freely available, the equilibrium final product price p must be given by the level of the minimum point q_s^m of the small firm's average cost curve. That is,

$$p = w + V^s(q_s^m)/q_s^m \equiv w + z. \tag{1}$$

(Given fixed proportions and uniform pricing by the monopolist, q_s^m and hence z are independent of w.) With output price parametric, the optimality conditions which determine the output q_l of a large firm are given by

$$p - \partial C^l/\partial q = p - w - V^l_q(q_l) = 0; \qquad V^l_{qq} > 0. \tag{2}$$

Equating industry supply to final product market demand $Q(p)$ determines the equilibrium number n_s of small firms; i.e.,

$$n_s q_s^m + \bar{n}_l q_l = Q(p). \tag{3}$$

This is the framework in which we shall examine the possibility of introducing a Pareto superior nonlinear outlay schedule for the monopolist's product.

3. The impossibility of a Pareto superior tariff

■ Following Willig, we assume that the initial uniform price w° charged by the monopolist is greater than marginal cost and that it is *undominated*; i.e., there does not exist any lower uniform price which yields as much profit for the monopolist. Willig constructed his Pareto superior outlay schedule by offering the largest consumer type a slight discount on the price of any *additional* units purchased. Any user with a downward sloping demand curve would avail himself of this offer, and since the additional units are sold at a (discount) price greater than marginal cost, the seller's profits increase. A portion of this gain can then be used to lower the price facing smaller consumers. In other words, the uniform price is replaced by a declining block tariff whose first block, equal in length to the initial demand of the largest user, has a price slightly below the initial uniform price. The price of the second or trailing block is set marginally lower than that of the first.

It should be clear that this algorithm cannot work in our model, because implicit in the logic of the above argument is the assumption that purchases by small consumers are unaffected by the discount offered to large buyers. However, since price, and thereby total quantity, in the final product market is determined by the costs of the small firms, the *total* quantity sold by the monopolist cannot increase as a result of the discount offered to large firms.

The increased revenues resulting from additional purchases by large firms are more than offset by the decrease in revenues resulting from the *exit* of some of the small firms from the industry. Offering the discount merely converts some high price sales into low price sales, resulting in lower profits. Lowering the price facing small firms would, of course, expand industry output; however, this would also lower profits since w^* was assumed to be undominated.

Not only does the Willig algorithm fail in our model, but it is also *impossible* to construct *any* nonlinear outlay schedule which is Pareto superior to an undominated uniform price. Any such tariff must be constructed so that in equilibrium: (a) the seller's profits increase; (b) the rents of large firms increase; and (c) the price facing final consumers decreases.

Although we shall relegate the mathematical proof of our impossibility result to the Appendix, the intuition is quite clear. A necessary (but not sufficient) precondition for satisfying (a) and (b) is that *total* producers' surplus must increase. Certainly no system of nonlinear transfer prices can do better (for producers) than a vertically integrated production monopoly. However, because of the fixed proportions assumption and the competitive structure of the downstream industry, *total industry costs* are being minimized at the initial equilibrium. Thus *given the output* $q^0 = Q(w^* + z)$, total producer profits are already at their maximum. Therefore, the only possibilities for increased profits must involve changes in q; but a decrease would result in a higher price, violating (c), while an increase must result in lower profits if w^0 was indeed undominated.

4. Concluding remarks

■ While, as noted by Willig, public utilities are most likely to be able to engage in nonlinear pricing, a sizeable portion of their output is sold to other firms which may compete with one another. Since we have demonstrated that Pareto improvements are not always possible in such circumstances, the implications of Willig's result for regulatory policy are somewhat unclear. A uniform price above marginal cost (e.g., equal to average cost) may be Pareto efficient, given available policy instruments. In the case of unregulated industries, the proscription of nonlinear pricing in the Robinson-Patman Act can be better rationalized once it is recognized that Pareto improvements are *not* always possible. In our model the gains of large customers can only come at the expense of the seller, final consumers, or smaller rivals. Losses to the latter not only damage the competitors, but may also diminish the vigor of competition. Indeed, this was the argument adduced in support of the "secondary line injury" provisions of the Act.[2]

Appendix

■ **Impossibility proof:** Letting $\pi_m(q)$ and $C_m(q)$ denote the profits and costs of a *uniform-pricing* monopolist expressed as functions of quantity, (1) yields

$$\pi_m(q) = [Q^{-1}(q) - z]q - C_m(q). \tag{A1}$$

The total costs of an optimized downstream production sector result from solving the program

[2] See, for example, Areeda (1974, pp. 866–867).

$$C_d(q) = \min_{q_i} z(q - \bar{n}_t q_i) + \bar{n}_t V^i(q_i). \tag{A2}$$

By the envelope theorem, $\partial C_d/\partial q = z$, a constant, whenever the freely available technology is used. The profits of a vertically integrated monopoly producer would be

$$\hat{\pi}(q) = qQ^{-1}(q) - C_m(q) - C_d(q). \tag{A3}$$

Since, given fixed proportions, the competitive industry modelled by (1)–(3) solves (A2), *total* producer profits at q^0 equal $\hat{\pi}(q^0)$. Now suppose there exists a $q' > q^0$ such that $\hat{\pi}(q') \geq \hat{\pi}(q^0)$. That is,

$$q'Q^{-1}(q') - C_m(q') - C_d(q') \geq q^0Q^{-1}(q^0) - C_m(q^0) - C_d(q^0). \tag{A4}$$

Since $C_d(q') - C_d(q^0) = z(q' - q^0)$, (A4) can be rewritten as

$$\pi_m(q') = [Q^{-1}(q') - z]q' - C_m(q') \geq [Q^{-1}(q^0) - z]q^0 - C_m(q^0) = \pi_m(q^0). \tag{A5}$$

But (A5) contradicts the hypothesis that w^0 (and therefore q^0) represented an undominated initial position. *Q.E.D.*

References

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