TECHNICAL APPENDIX C

Description of Logistic Growth Variable

Logistic growth is modelled as follows:

$$\frac{\alpha}{[1 + \beta*EXP(-\delta*T)]} + 10000*(\alpha - ABS(\alpha)) + 10000*(\beta - ABS(\beta)) + 10000*(\delta - ABS(\delta))$$

where α, β, and δ are the parameters to be estimated, EXP is the symbol for exponential, ABS is the symbol for absolute value, and T indicates time. The parameter α represents the maximum adoption level, the parameter β represents the time it takes to reach the maximum adoption level, and the parameter δ reflects the rate of adoption. The rate of change of the dependent variable with respect to time is proportional to the current level of the dependent variable and also to the distance remaining to reach the maximum adoption level α. The parameters α, β, and δ must all be positive. The terms $1000*(\alpha-ABS(\alpha))$, $1000*(\beta-ABS(\beta))$, and $1000*(\delta-ABS(\delta))$ are called the penalty functions. These functions vanish when convergence is attained and are used to ensure that the convergence occurs such that the positivity conditions hold.

This is a nonlinear expression and needs to be estimated using a nonlinear estimation technique. In practice this is handled in two stages. In the first stage, using a nonlinear least squares technique, an equation is estimated with a logistic trend term. From this equation the parameters of the logistic component of the model are used to construct the market penetration variable called the Z-variable.

The variable is simply the prediction from the equation using only the logistic component.

In the second stage the coefficient of the computed Z-variable is constrained to equal 1. This is achieved by subtracting the Z-variable from the dependent variable to obtain a new transformed dependent variable. The estimation then proceeds as usual but with the new transformed dependent variable. The final forecasts are retransformed by adding the Z-variable back to the forecasts from the model.

The statistical calculations are performed as a standard feature in the computer software system, "Regression Analysis of Time Series". They are invoked by using the "NONLIN" and "NLLS" commands producing the non-linear estimations via non-linear least squares as discussed above.