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Institutional Cost Contribution Requirement for Competitive Products — Docket No. RM2017-1

ADDITIONAL DECLARATION OF SOILIOU DAW NAMORO
FOR THE PUBLIC REPRESENTATIVE

(September 12, 2018)

I. Autobiographical Sketch

My name is Soiliou Daw Namoro. I am an economist with the Postal Regulatory Commission (PRC), where I have been working since 2017. I hold a Ph.D. in Economics (State University of New York, at Stony Brook) and a Ph.D. in Statistics (Catholic University of Louvain, Louvain-la-Neuve, Belgium).

Before joining the PRC, I worked for more than a decade as a lecturer at the University of Pittsburgh, Department of Economics, where I taught graduate-level courses of Econometrics and Mathematics. I have sat on several Ph.D. thesis committees. I also taught several undergraduate courses, such as Industrial Organization, Health Economics, Development Economics, and Financial Markets and other similar courses. I have conducted research in theoretical and applied econometrics, applied microeconomics, health economics, and development economics. I have published research articles in peer-review Journals, such as the Journal of Econometrics and the Journal of Economics. I have contributed to books. The latest contribution is co-authored with Margaret Cigno, Director of the Office of Accountability and Compliance at the PRC and published in The Contribution of the

Postal and Delivery Sector: Between E-Commerce and E-Substitution (Topics in Regulatory Economics and Policy).

II. Purpose of Declaration

The Public Representative tasked me with reviewing, analyzing and preparing comments on my findings regarding the Commission's formula proposed in Order No. 4742 to calculate the minimum contribution requirement to total institutional costs by the Postal Service's competitive products.

III. Declaration

1. Introduction and Outline

Upon its review of the Comments raised to the formula-based approach to setting the appropriate share proposed in Order No. 4402[1], the Commission's has proposed a new equation to calculate the annual minimum contribution share by competitive products to institutional costs. As its predecessor proposed in Order No. 4402, henceforth referred to, for convenience, as the Order 4402 formula, the new equation recursively updates the minimum contribution share computed for a given year to set the minimum share for the following year. Using the minimum appropriate share determined in 2006 of 5.5 percent for FY 2007 and FY 2008, the Commission starts recalculation from FY 2009.

The content of the informational materials, specifically, the data and the statutory guidelines, that underlie the new formula, is not substantially different from the materials that were used by the Commission to develop and support the Order 4402 formula. Consequently, although, as I will discuss later in the declaration, the new formula is, in many respects, an improved version of the Order 4402 formula, the realized

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[1] Notice of Proposed Rulemaking to Evaluate the Institutional Cost Contribution Requirement for Competitive Products, February 8, 2018 (Order No. 4402).

improvement remains short of demonstrating that the ongoing appropriate share has become inappropriate and, therefore, needs to be changed. Further, recent developments in the delivery industry suggest that a careful review of the Commission's appropriate share regulation involves judgments which may not be accurately depicted in a mathematical formula.[2] The last statement should be qualified, however, by the possibility for the Commission to revisit the proposed formula at any time in the future. I also note the ability of the formula to absorb the most catastrophic down-turn of the market from the Postal Service's perspective, by reducing the appropriate share to the stable value of zero until a new non-zero appropriate share is set by the Commission. This point will be further developed in the part of the declaration that discusses the stability of the formula.

The purpose of this declaration is to evaluate the new formula. This evaluation is guided by the precept that the formula must enable the Commission to decide about the appropriate share based on incomplete information and in the presence of unforeseen events. It consequently focuses on the following points:

The Reasoning Behind the new formula.
Are there other implicit reasons than those presented by the Commission to design the formula that need to be made explicit? Does this reasoning provide additional ground for using the formula as a decision-making tool suited to its intended use?
The stability of the formula: *how likely is the formula to generate shares that change dramatically with unpredictable time patterns? How likely is it to produce long-lasting and self-sustaining trends of shares, which ultimately end in one of the two extreme values of the shares, 0 and 1?*

These points are informally developed as follows:

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[2] The complexity of the involved judgments is not unlike the one regarding pricing described in Docket R87-1, *Opinion and Recommended Decision*, March 4, 1988, Vol 1, at 360. Noticeable in this respect is the fact reported by the Wall Street Journal that Amazon is "inviting entrepreneurs to form small delivery in a continued quest to build a vast freight and parcel shipping network." This move by the online retail giant further strengthens its position as a strategic player in the delivery market with large monopsonistic powers. The evolution could, if it amplifies, have a major impact on the development of the structure of the package delivery industry, and increase market uncertainty, a challenge already faced by the Postal Service, whose financial sustainability continues to erode, according to the Commission's financial analysis for the fiscal year 2017. See PRC, *FINANCIAL ANALYSIS of United States Postal Service Financial Results and 10-K Statement*, Fiscal Year 2017, April 5, 2018, at 6.

- Much intuition can be gained about the new formula if one takes the Commission's *Competitive Contribution Margin* (CCM) for what it effectively is: the percentage of gross sales revenue available to cover (or contribute to) institutional costs. Hence, changes in this ratio, or in its absolute counterpart, the numerator of the CCM, measure in relative and absolute terms, possibly imperfectly, *the joint ability of the Postal Service's competitive products to contribute to the coverage of institutional costs.* I refer, in short, to this capability as the *Postal Service's ability to pay*. Based on this interpretation of the CCM, I establish that the new formula's updating rule can be summarized as follows:

  Raise (reduce) the minimum share for the next fiscal year if the Postal Service's ability to pay has increased (decreased) faster than a reference baseline this year, compared to the last. This rule is shown in two alternative but equivalent versions. While the rule is intuitively sound, it nevertheless raises an important question regarding the principles guiding the Commission's minimum-share updating policy:

Should any relative increase in the Postal Service's ability to pay be automatically construed as an increase in its obligation to pay? A similar question can, of course, be asked about relative decreases in the Postal Service's ability to pay.

To the best of my reading of Order No. 4742, these questions are not addressed therein, but they essentially concern the degree of flexibility that the Postal Service has, or ought to have, in conducting its business within its statutory scope.

- The fear that the formula could generate patterns of shares that change dramatically over time is among the reasons for worrying about stability. However, the word "stability" applied to a recursive discrete-time, or to a differential equation, has several different meanings and the omission to properly define the sense in which one uses this word may generate confusion where the policy debate needs clarity. For example, the popular notion of *Lyapunov stability* applies to equilibrium points of the recursion. In the present case, there is a unique equilibrium, the zero share, and stability of this equilibrium point means the wandering of the computed shares towards zero. Some stakeholders may find the property very attractive if it holds. Others may dislike it very much. As it pertains to the Commission's formula, *Lyapunov stability* cannot be proved or disproved theoretically without making questionable assumptions on the dynamics of the Postal Service's ability to pay. Because, in the present case, any formal proof of stability – or lack of stability – cannot avoid relying on unverifiable assumptions on the likely time path of the Postal Service's ability to pay, I provide simulation results based on the assumption that the rate of change in the share can only take the lowest or highest value observed over the period 2007-2017.

Regarding the likely behavior of the computed shares in the long run, I discuss the meaning of the eventual convergence of the shares to a limit when the time tends to infinity. Intuitively, this simply means that the shares eventually become arbitrarily close to one another. I provide necessary conditions, as well as sufficient conditions, for the convergence. These conditions show that, here also, convergence cannot be proved or disproved without making questionable assumptions on the dynamics of the Postal Service's ability to pay.

As for the possibility of unpredictable erratic trends in the computed shares, I stress the fact that the indexation – in the formula – of the change in the appropriate share to the change in the Postal Service's ability to pay implies that the computed shares and the Postal Service's ability have *exactly* the same time-trends. Hence, predicting the time-trends in the first amounts to predicting the trends in the second. I discuss the latter trends by focusing on product-specific price-to-unit-cost ratios and the price ratios between the Postal Service and its competitors.

2. Evaluation of the New Formula

The Commission's new formula is stated as

$$AS_{t+1} = AS_t * (1 + \%\Delta CCM_{t-1} + CGD_{t-1}) \qquad (1)$$

$$If\ t = 0 = FY\ 2007,\ AS = 5.5\%$$

Where,

AS = Appropriate Share
CCM = Competitive Contribution Margin
CGD = Competitive Growth Differential
t = Fiscal Year

Two main changes have been made by the Commission to its previous Order 4402 formula: the first – and in my view the most important – is the substitution of the *Competitive Growth Differential* (CGD) for the previous growth rate of the *Competitive Market Output* (CMO). The competitive Growth Differential is determined as

$$CGD = Competitive\ Growth\ Differential = Market\ Share_{USPS} * (\%\Delta Revenue_{USPS} - \%\Delta Revenue_{C\&M}), \qquad (2)$$

where the revenue data are first adjusted for inflation using the Consumer Price Index for All Urban Consumers (CPI-U) as the deflator.[3] This modification shields the updating mechanism from some rather counter-intuitive behaviors. For example, the trends in the Postal Service's competitors' revenue over a fiscal year, which can represent pure pricing trends, can no longer translate in absolute terms– as they could

[3] In a continuous-time analysis, where the time runs over an interval, say [a, b], with a>0, and a<b, and the rate of change in a variable *X* is defined as the logarithmic derivative $r_X(t) = \frac{\frac{\partial(X)}{\partial(t)}}{X(t)}$, no adjustment of the revenue data for inflation would be necessary. This is because the difference between the two growth rates would be exactly equal to the growth rate of their ratio. The ratio is scale-invariant in that if both revenues are scaled up or down by the same constant, for example, by the price index, it remains unchanged. In the discrete-time framework, the same result can be achieved by simply computing the growth rates as first differences of natural logarithms. See for example, Miquel Faig, "Seasonal Fluctuations and the Demand for Money," *The Quarterly Journal of Economics,* Vol. 104, No. 4 (Nov., 1989), pp. 847-861. Page 851.
For example, the growth rate of a revenue *R* over the period [t-1, t] is computed as $ln(R_t) - ln(R_{t-1})$. So, the Commission could avoid using any revenue deflator by using this well-known discrete-time approximation to the computation of growth rates of positive variables. I illustrate the methods with the data provided along with Order No. 4742 in the appendix to this declaration. This remark may be important at least in one respect: using the suggested computational method would eliminates any potential disputes about the most appropriate price index to use.

in the Order 4402 formula – into changes to the next fiscal year's minimum share.[4] In the new formula they can do so only differentially, relative to the trends in the Postal Service's own competitive revenue.

The second change made to the Order 4402 formula is the substitution of total competitive attributable costs for total competitive volume-variable costs in the Postal Service Lerner Index (PSLI).[5] The resulting new ratio is referred to as the *Competitive Contribution Margin* and formally defined as[6]

$$Competitive\ Contribution\ Margin = \frac{Total\ Revenue - Total\ Attributable\ Cost}{Total\ Revenue} \quad (3)$$

To proceed with the assessment of the resulting new formula, I will make use of the following notations and elementary algebraic equalities. These are essentially the same notations that were already used in my declaration about the Order 4402 formula, with a few changes to account for the variables that are present in the new formula.

2.1. Notations

$R_{Po}(t)$: Postal Service's competitive-product revenue in FY *t.*

$R_{co}(t)$ Competitors' (Couriers and Messengers) revenue in FY *t*.

$CAC(t)$: Total competitive attributable costs.

$r_X(t) = \frac{X(t)-X(t-1)}{X(t-1)}$: The rate of change in the variable *X,* over the time interval.

[t-1, t]. For example, $r_{R_{Po}}(t)$ denotes the rate of change (or the percentage change, if multiplied by 100) in the Postal Service's competitive revenue over *[t-1, t]*.[7]

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[4] This issue is discussed extensively in my declaration about the Order 4402 formula. See Declaration of Soiliou Daw Namoro for the Public Representative, April 16, 2018 (Namoro Decl.) Section 3.

[5] The per-piece version of the revenue and the costs is also abandoned in the crafting of the new formula. The division of revenue and cost by the total competitive volume was shown in my declaration about the Order 4402 formula to be unnecessary for the computation of the Postal Lerner index. Namoro Decl. Section 2.2.

[6] It might be worth noting that with this change, the updating of the appropriate share explicitly depends to some extent on past realized contributions. A possible issue raised by this dependence is the risk of ratchet effect, which was discussed in my declaration about the formula 4402-formula. See Namoro Decl. at 21.

[7] Later in the declaration, the expressions "rate of change" and "percentage change" are used interchangeably, with a precision when a number is in percent.

$r_{Po}(t)$ = growth rate of $R_{Po}(t)$, $r_{co}(t)$ = growth rate of $R_{co}(t)$

$\pi(t) = R_{Po}(t) - CAC(t)$*:* Postal Service's net competitive revenue in FY *t* (net of total competitive attributable costs)

$CMO(t) = R_{R_{Po}}(t) + R_{co}(t)$: Total revenue on the competitive market (competitive market output) in FY *t.*
$S(t) = \frac{R_{Po}(t)}{R_{Po}(t) + R_{co}(t)}$: Postal Service's share of competitive revenue in FY *t* (its market share)
$CCM(t)$: Postal Service Competitive Contribution Margin in FY *t*.
$R_{Po,i}(t)$ = the Postal Service's sales revenue from its Product *i.* Corresponding rate of change at time *t*: $r_{Po,i}\,(t)$.
$Q_{Po,i}(t$ = the Postal Service's volume of its Product *i.* Corresponding rate of change at time *t*: $r_{QPo,i}\,(t)$.
$CAC_i(t)$ = Postal Service's product-*i's* total attributable cost. Corresponding rate of change at time *t*: $r_{CAC_i}\,(t)$.
$a_i(t) = \frac{CAC_i(t)}{Q_{Po,i}(t)}$ = Product *i's* per-piece competitive attributable variable cost.
$p_{Po,i}(t)$ = the Postal Service's unit price for its product *i.*
$R_{co,i}(t)$ = Competitors' sales revenue from Product *i.*
$p_{co,i}(t)$ = Competitors' unit price for product *i.*

In the new notations, $\%\Delta CCM_{t-1}$ becomes $r_{CCM}(t)$ and CGD_{t-1} becomes, by definition,

$S(t-1)[r_{Po}(t) - r_{co}(t)]$.

The recursive equation (1) can now be restated as

$$AS_{t+1} = AS_t\{1 + r_{CCM}(t) + S(t-1)[r_{Po}(t) - r_{co}(t)]\} \quad (4)$$

$$AS_{2008} = AS_{2007} = 0.055$$

Or , in short,

$$AS_{t+1} = AS_t\big(1 + r(t)\big)\,, \quad (4\ bis)$$

with $r(t)$ denoting the rate $r_{CCM}(t) + S(t-1)[r_{Po}(t) - r_{co}(t)]$.[8]

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[8] The possibility that the trends in the revenues and costs are subject to random shocks may suggest that a better strategy for assessing the formula is to account for these shocks by treating the rate $r(t)$ as random in the equation $AS_{t+1} = AS_t\big(1 + r(t)\big)$. Assuming that

(i) for each *t,* $r(t)$ has a finite mathematical expectation (conditional on time-*t-1*'s variables), which is expressed as $E(r(t) \mid t-1) = \rho(t)$, where *E(. |.)* denotes the conditional expectation operator, and

Likewise, the defining relation (3), see page 7, is more precisely stated as

$$CCM(t) = \frac{R_{Po}(t) - CAC(t)}{R_{Po}(t)} \qquad (5)$$

I make use of the following algebraic equalities and their corresponding approximations.

If two variables, say *X* and *Y*, are observed in discrete time *t,* then one has

$$r_{XY}(t) = r_X(t) + r_Y(t) + r_X(t) r_Y(t) \qquad (6)$$

$$r_{X/Y}(t) = r_X(t) - \left(\frac{r_Y(t)}{1+r_Y(t)}\right) - r_X(t)\left(\frac{r_Y(t)}{1+r_Y(t)}\right) \qquad (7)$$

$$r_{X \pm Y}(t) = \left(\frac{X(t-1)}{X(t-1) \pm Y(t-1)}\right) r_X(t) \pm \left(\frac{Y(t-1)}{X(t-1) \pm Y(t-1)}\right) r_Y(t) \qquad (8)$$

From the approximation $\ln(1+r) \approx r$, which holds for "small" real numbers r , the relations (5) and (6) can, in turn, be approximated, when $r_X(t)$ and $r_Y(t)$ are "small", as[9]

$$r_{XY}(t) = r_X(t) + r_Y(t) \qquad (9)$$

$$r_{X/Y}(t) = r_X(t) - r_X(t) \qquad (10)$$

2.2. The Reasoning Behind the New Formula

2.2.1. The Competitive Contribution Margin Reinterpreted

The Commission uses the Competitive Contribution Margin (CCM) to measure the Postal Service's market power on the competitive market. As a measure of market power, the new index is not substantially different from its predecessor, the Postal

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(ii) the initial share is not random – the Commission simply sets it to some appropriate level, e.g., 5.5% – the computed shares can be thought of as obeying the mean recursive equation $AS_{t+1} = AS_t\big(1 + \rho(t)\big)$, where all involved growth rates are correspondingly replaced by their (conditionally) expected growth rates. Hence, in the presence of random shocks, every statement on the computed share AS_t can simply be understood as a statement on its expected value, given the data pertaining to the fiscal year *t-1*. See also the footnote 25 on page 18.

[9] The relations (5) and (6) are proved in my declaration about the 4402-formula. Namoro Decl. at 11. Using the proofs therein, the approximations (8) and (9) follow from the properties of the logarithm function. The word "small" is written in quotation marks because I do not define it rigorously. In a continuous-time analysis, the relations (9) and (10) are always exact.

Service Lerner index (PSLI). Hence, my remarks about this use of the CCM are the same as the ones I formulated about the PSLI in my declaration about the Order 4402 formula.[10]

In fact, the CCM and its predecessor, the PSLI, are more akin to the *Contribution Margin Ratio*, when computed for a group or a class of products. The contribution margin ratio is, for a group of products, a formula that calculates the ratio of the contribution margin– defined as total sales minus total variable costs– to total sales.

$$Contribution\ Margin\ Ratio = \frac{Total\ Sales - Total\ variable\ Costs}{Total\ Sales} = \frac{Contribution\ Margin}{Total\ Sales} \quad (11)$$

This ratio is widely used in managerial accounting where it is also referred to, sometimes, as the *contribution sales ratio*.[11] The difference between the CCM and the contribution margin ratio is the use of total attributable cost in the former, where total variable costs is used in the latter. Both ratios measure the percentage of sales dollars available to cover (or contribute to) fixed costs, where the fixed costs can, in the present context, be extended to institutional costs. This interpretation of the ratio has not much to do, if anything, with the notion of market power, although one may find reasons to connect the two in some particular theoretical framework.

This kind of ratio is, in fact, not absent from the Postal Service past reporting practices, where it has been computed for a class of products as [12]

$$\frac{Total\ Sales - Total\ Volume\ Variable\ Costs}{Total\ Sales} \qquad (12)$$

For example, in the table with the title *Summary of Revenue and Cost For Major Service Categories*, exhibit USPS-11C of Fiscal year 2000 Cost and Revenue Analysis,[13] the numbers in the column labelled *Contribution Margin* are determined at the classes and sub-classes levels of mail using the ratio (12). At product specific level, revenue per piece and marginal cost are

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[10] See Namoro Decl. at 7-9.
[11] See, for example, Davis, C. E. and E. Davis, *Managerial Accounting,* Wiley, 2012, at 65.
[12] I note here that if marginal cost does not vary with volume, then the expression (12) is exactly equal to the contribution margin ratio.
[13] Docket R2000-1. Direct testimony of Karen Meehan on Behalf of the United States Postal Service. (Meehan 2000).

substituted, respectively, for the Total Revenue and Total Volume-Variable Cost. The indicator is also reported in absolute terms, i.e., without dividing by the revenue or by the revenue per piece. The absolute version is referred to as the *Contribution Margin* and the percentage form, as the *Contribution Margin Percent*, with the following description:

"The relation of unit revenue to marginal cost provides a measure of the contribution earned by a subclass or mail category at the margin of production. The measure Indicates the rate at which a given subclass offsets all other costs of the Postal Service. The contribution margin percent column gives the same information as a proportion of revenue per piece."[14]

This reporting practice is also found in the Postal Service's Cost and Revenue Analysis for the Fiscal Years 2006 and 2007.

An Increase in the CCM is clearly desirable *if* the gross revenue also increases.[15] A decrease in the CCM is undesirable *if* gross revenues also decreases. This is so because the former (the latter) means that a larger (a lower) percentage of – the larger (the lower) – gross sales revenue is available to pay, for example, for institutional costs. So, increases in the CCM should be taken for what they really are: they represent increases in *the competitive products' joint ability to cover institutional costs.*[16] I refer, in short, to this capability as the Postal Service's *ability to pay.*

This interpretation will be used to provide several intuitions for the updating mechanism built into the new formula. To keep the analytical presentation simple, I will constantly confine myself to the use of the equalities (8), (9), and (10). Although the last two are only approximative, the numerical difference from using the exact equalities (6) and (7) is practically negligible within the present context. Where the difference may matter, I will make the necessary precisions. Hence, I formally write equality for approximate equality in the development.

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[14] Meehan 2000. Exhibit USPS-11C Cost and Revenue Analysis Base Year 1998. The expression "all other costs" in the citation is imprecise and should be understood as "all volume-variable costs".

[15] If gross revenue has decreased, the statement is no longer necessarily true. As I remarked in my declaration about the Order 4402 formula, a decrease (an increase) by the same amount in both the revenue and the cost will lead to a counter-intuitive increase (decrease) in the Postal Service's Lerner Index. These remarks equally apply to the CCM.

[16] Decreases in the CCM are decreases in the competitive products' joint ability to cover institutional costs.

I begin by making the following remark about the equality (4): the growth rate of the appropriate share, $r_{CCM}(t)+\ S(t-1)[r_{Po}(t)-r_{co}(t)]$, is equal to the growth rate of the expression[17]

$$J_0 \coloneqq \left(\frac{R_{Po}(t)-CAC(t)}{R_{Po}(t)}\right)\left(1+\frac{R_{Po}(t)}{R_{co}(t)}\right)=CCM(t).\left(1+\frac{R_{Po}(t)}{R_{co}(t)}\right) \qquad (13)$$

Indeed, using equality (9), the growth rate of J_0 is derived as

$r_{J_0}(t)=r_{CCM}(t)+r$, where r is the growth rate of the expression $\left(1+\frac{R_{Po}(t)}{R_{co}(t)}\right)$, which is derived, using (8) and (9) and (10) as

$$r=\left(\frac{\frac{R_{Po}(t-1)}{R_{co}(t-1)}}{1+\frac{R_{Po}(t-1)}{R_{co}(t-1)}}\right)\left(r_{Po}(t)-r_{co}(t)\right)=\ S(t-1)\left(r_{Po}(t)-r_{co}(t)\right).$$

Hence, $r_{J_0}(t)=r_{CCM}(t)+(t-1)\left(r_{Po}(t)-r_{co}(t)\right)=r(t),$ $\qquad (14)$

as claimed.

My second remark is that the expression J_0 has the following alternative forms:

$$J_0=\frac{CCM(t)}{1-S(t-1)}:=J_1 \qquad (15)$$

and

$$J_0=\frac{R_{Po}(t)-CAC(t)}{\frac{1}{2}\left(\frac{1}{2}\left[\frac{1}{R_{Po}(t)}+\frac{1}{R_{co}(t)}\right]\right)^{-1}}=\frac{R_{Po}(t)-CAC(t)}{\frac{1}{2}H\left(\ R_{Po}(t);R_{co}(t)\right)}\coloneqq J_2 \qquad (16)$$

where $H\left(\ R_{Po}(t);R_{co}(t)\right)$ denotes the simple harmonic mean of the two revenues $R_{Po}(t)$ and $R_{co}(t)$.[18] The equalities (15) and (16) are proved in the Appendix. I interpret

[17] In the declaration, the sign ":="stands for "denotes" or "is denoted by", depending on the side of the sign where the new notation appears. Hence, if *A* is a well-defined expression and *j0* is a new notation, "*j0 := A*" means "*j0* denotes the expression *A*", and "*A :=j0*" means "the expression *A* is denoted by *j0*".

[18] The simple harmonic mean of two positive numbers, say *a* and *b,* is defined by the formula $\left[\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\right]^{-1}$.

them by noting that J_1 and J_2 are ratios and a ratio of two positive numbers increases to become a ratio of two other positive numbers if and only if the rate of change in the numerator exceeds that of the denominator.[19]

Focusing on J_1first, the relations (4) and (14) imply, therefore, that

the appropriate share for the next fiscal year is raised if and only if the Postal Service's ability to pay has increased fast enough over the current fiscal year, where "fast enough" means faster than competitor's revenue, the latter evaluated relatively to total market revenue.[20]

Formally, the increase in the appropriate share occurs if and only if $r_{CCM}(t) > r_{1-s}(t)$.

To interpret the expression J_2, I first note that the harmonic mean of the two revenues, $H\left(R_{Po}(t); R_{co}(t)\right)$, could be replaced by other means, for example, the usual average, i.e., the arithmetic mean, or the geometric mean, etc. I indicate in the appendix, how the rate $r(t)$ slightly changes with the changing of the type of the mean used at the denominator of J_2. The simple harmonic means has the property that it is biased towards the smallest of the two numbers, which, in this case, is the Postal Service's competitive revenue, i.e., $R_{Po}(t)$. So, $H\left(R_{Po}(t); R_{co}(t)\right)$ measures the market's average revenue per firm, albeit with a greater weight implicitly given to the Postal Service's revenue than its competitors'. I refer to $H\left(R_{Po}(t); R_{co}(t)\right)$, simply as the *industry average revenue*.

With these preliminaries, the interpretation of the expression J_2 almost mirrors that of J_1. Specifically, the relations (4) and (14) imply, here also, that

the appropriate share for the next fiscal year is raised if and only if the Postal Service's ability to pay – in absolute form– has increased fast enough over the current fiscal year, where "fast enough" means faster than the industry average revenue.

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[19] Indeed, if *A* and *B* are these two positive numbers and *A* changes to *A+a,* while *B* changes to *B+b,* such that *A+a>0* and *B+b>0,* then $\frac{A+a}{B+b} > \frac{A}{B}$ is equivalent to $\frac{A+a}{A} > \frac{B+b}{B}$, which, in turn, is equivalent to $\frac{A+a}{A} - 1 > \frac{B+b}{B} - 1$.

[20] In other words, faster than competitor's market share.

Formally, the increase in the appropriate share occurs if and only if $r_{CCMA}(t) > r_H(t)$, where $r_{CCMA}(t)$ denotes the rate of change in the (absolute) CCM, and $r_H(t)$ denotes the rate of change in the industry average revenue.[21]

Returning to the related questions that were asked in the introduction,

2.2.2. Are there other implicit reasons than those presented by the Commission to design the formula that need to be made explicit?

(i) The new formula models the joint ability of the Postal Service's competitive products to cover institutional costs –in short, the Postal Service's ability to pay– both in absolute and in relative terms.

(ii) It compares the changes in the Postal Service's ability to pay to appropriate referential baselines. The changes in the Postal Service's *absolute* ability to pay is compared to changes in the industry average revenue, which is also an *absolute* measure. The changes in the Postal Service's *relative* ability to pay is compared to changes in the competitors' share of total market revenue, which is also a relative measure.

Since the apparent purpose of the new formula is to index the change in the appropriate share to the change in the Postal Service's ability to pay, its underlying modeling strategy and the associated decision rule are, in my view, logically sound.

2.2.3. Does this reasoning provide enough ground for using the formula as a decision-making tool suitable to its intended use?

To answer this question, I find it useful to revisit the *contribution margin* and the *contribution margin ratio*. The CCM, just as the contribution margin, is a piece of accounting information, which, together with other accounting and economic data, helps companies– in the present case, the Postal Service – measure their operating leverage. The contribution margin, in fact, indicates the amount from sales that is available to cover fixed costs *and contribute to profit*. Likewise, the CCM (absolute and relative) should indicate the amount from (respectively the percentage of) sales that is

[21] I note here that, by the relation (6) or (9), the ½ multiplying the harmonic mean does not affect its growth rate.

available to the Postal Service to cover its institutional costs *and other financial needs.* The flexibility that it has, or ought to have, in allocating the CCM should be a factor to consider by the Commission in deciding about the minimum share. These considerations raise the question stated in the introduction regarding the principles guiding the Commission's minimum-share updating policy. I believe that it is worth repeating and emphasizing this question: *Should any relative increase (or any relative decrease) in the Postal Service's ability to pay be automatically construed as an increase (respectively, a decrease) in its obligation to pay?*

The answer to the section's title-question is, in my view, contingent on the answer to the last question. The new formula is suitable to be used as an updating rule of the appropriate share only if the Commission's guiding principle is that all relative increases (or decreases) in the Postal Service's ability to pay are to be considered *in full* as increases (respectively, decreases) in its obligation to pay. However, the precept that the Commission is seeking to determine the appropriate share as a *minimum,* should lead it to take into consideration in its decision to increase or lower the appropriate share, the existence of other financial needs that the Postal Service's contribution margin could cover. The complexity of the judgments required by this consideration is less amenable to a simple mathematical updating rule as the one implied by the new formula.

2.3. Stability of the New Recursive Equation

2.3.1. Overview

To study the stability of the new formula, Equation (4) will be written in the cumulative product form as

$$AS_{t+1} = AS_0 \prod_{j=0}^{j=t}\{1 + r(j)\} \qquad (17)$$

where the initial time is conveniently relabeled as zero and

$$r(j) \coloneqq r_{CCM}(j) + S(j-1)[r_{Po}(j) - r_{co}(j)] \qquad (18)$$

This section has three objectives. The first is to define a popular notion of stability – *Lyapunov stability* – and, more importantly, to show that in the context of the new formula, some of the conditions that insure this property may be undesirable. Specifically, I first remark the fact that (i) the zero minimum share is an equilibrium share and (ii) constant erosion of the Postal Service's ability to pay – relatively to the reference baselines – would guaranty the *Lyapunov* stability of this equilibrium share, which is not necessarily an attractive scenario.[22]

However, in regard of the prospect that the Commission is seeking to compute an *ex-ante* minimum share, i.e., a share that is a *minimum* before its enforcement as a *required minimum*, *Lyapunov* stability is – or may be – a desirable property, because it insures that if the initial share is close enough to zero, all subsequence shares will be wandering in the neighborhood of zero. In fact, in the present case, as I will soon show, *Lyapunov* stability of the equilibrium share has the necessary consequence that the computed shares will, in the long run become arbitrarily close to one another, i.e., converge to a limit that remains in the neighborhood of zero, provided that the initial share is set close enough to zero.

The second objective is to provide alternative sufficient conditions – conditions that are different from *Lyapunov* stability, but are by no means necessary – for the convergence of the computed shares to a limit when the time tends to infinity. The last objective is to discuss questions related to the possibility of unpredictable erratic trends in the computed shares.

2.3.2. Equilibrium Share, *Lyapunov* Stability, and Asymptotic Stability

"Stability" is a word primarily used in system theory, an interdisciplinary study of systems, where the word "system" designates any "complex of interacting elements".[23] In the present context, the new formula can be thought off as a mathematical description of a dynamic – or time-evolving – system.

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[22] In fact, they insure the convergence of the shares to zero.

[23] Von Bertalanffy, L. 1968. *General System theory: Foundations, Development, Applications*, New York: George Braziller, Page 55.

Among the many properties of a dynamic system, two are of special interest in the present context and pertain to the time-trends in the system's behavior. These are: *positive and negative feedback loops*.

"In a positive feedback loop, a trend gives rise to forces which increase the trend… In a negative feedback loop, a trend gives rise to counter-forces which hold it in check."[24]

A positive feedback loop would be present in the equation (4) if, for example, in every fiscal year the minimum share set by the formula does affect the Postal Service's ability to pay by increasing it, with the consequence that the share for the next fiscal year is set higher than the current. In the considered case, the calculated shares would exhibit a monotone increasing trend. In contrast, the new equation would be characterized by a negative feedback loop if, for example, the minimum share set for any given fiscal year has a negative impact on the Postal Service's ability to pay– for example by forcing it to raise its prices and loose market to its competitors – and, therefore, leads to a lower minimum share calculated for the next fiscal year.[25] In the latter case, the share sequence produced by the equation would exhibit a monotone decreasing trend.

More insight can be gained on possible sources of positive or negative feedback loop within the context of the new formula by expressing the rate $r_{CCM}(t)$ and the corresponding representation of $r(t)$ in more convenient forms. Using (10) and (8), one has

$$r_{CCM}(t) = r_{(R_{Po}-CAC)}(t) - r_{Po}(t)$$

$$= \left(\frac{R_{Po}}{R_{Po}-CAC}\right)_{t-1} r_{Po}(t) - \left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1} r_{CAC}(t) - r_{Po}(t) \quad (19)$$

$$= \left(\frac{R_{Po}}{R_{Po}-CAC} - 1\right)_{t-1} r_{Po}(t) - \left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1} r_{CAC}(t) \quad (19\ bis)$$

$$= \left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1} [r_{Po}(t) - r_{CAC}(t)] \quad (20)$$

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[24] These definitions are from Dietrich Fischer, "Peace as a self-regulating process," in *Handbook of Peace and Conflict Studies,* Edited by Charles Webel and Johan Galtung, Chapter 13, Routeledge, 2007, at 189.

[25] Some may sustain that a higher minimum share, by forcing the Postal Service to raise its prices, would lead competitors to also raise their prices. The process may, in this case, ultimately lead to a sustained increase in the Postal Service's ability to pay, as well as in its competitors' revenues. Of course, other interests than the Postal Service's and its competitors' are to be taken into account in this scenario.

where $\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1}$ is written as a shorthand for $\left(\frac{CAC(t-1)}{R_{Po}(t-1)-CAC(t-1)}\right)$. The rate of change $r(t)$ can, consequently, be written as

$$r(t)=\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1}[r_{Po}(t)-r_{CAC}(t)]+\ S(t-1)[r_{Po}(t)-r_{co}(t)] \qquad (21)$$

or

$$r(t)=\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1} r_{\left(\frac{R_{Po}}{CAC}\right)}(t)+\ S(t-1)r_{\left(\frac{R_{Po}}{R_{co}}\right)}(t) \qquad (22)$$ [26]

From (21) and (22), it is clear that the main drivers of the change in the computed share from Fiscal Year *t* to Fiscal Year *t+1* are the two rates $r_{\left(\frac{R_{Po}}{CAC}\right)}(t)=[r_{Po}(t)-r_{CAC}(t)]$ and $r_{\left(\frac{R_{Po}}{R_{co}}\right)}(t)=[r_{Po}(t)-r_{co}(t)]$, i.e. the rate of change in the Postal Service's revenue-to-cost ratio,[27] and the rate of change in the ratio $\frac{R_{Po}}{R_{co}}$, where the latter can be put in one-to-one correspondence with the Postal Service's market share.[28] So, the changes in the calculated shares are mainly driven by the changes in the Postal Service's competitive revenue-to-cost ratio and the changes in its market share.

Assuming that costs are only negligibly affected by the level of the minimum share, and assuming zero inflation, a positive feedback loop would mean that a higher computed share for a fiscal year does – presumably – accelerate the increase in the Postal Service's competitive revenues, and/or raise the rate of increase above that of its competitors' revenue. If this scenario is to be realized through some strategic pricing behavior by the Postal Service, then its likelihood will strongly depend on implicit assumptions regarding the demand elasticities of the Postal Service's products, as well as competitors' strategic reaction to its pricing behavior. In other words, very strong

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[26] Note that when random shocks on revenues and costs are assumed, as discussed in the footnote 8 on page 9, one has $E(r(t)\,|t-1)=\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1}[E(r_{Po}(t)|t-1)-E(r_{CAC}(t)|t-1)]+\ S(t-1)[E(r_{Po}(t)|t-1)-E(r_{co}(t)|t-1)]$. Hence, one just needs to think of every growth rate as an expected growth rate.

[27] More precisely, its competitive revenue-to-attributable-cost ratio.

[28] More precisely, "with its revenue share". If *s* denotes the market share, the one-to-one correspondence is given by the equality $\frac{R_{Po}}{R_{co}}=\frac{s}{1-s}$, which can be checked by substituting $\frac{R_{Po}}{R_{Po}+R_{co}}$ for *s* in the right-hand side. The inverse equality is $s=\frac{m}{1+m}$, where $m=\frac{R_{Po}}{R_{co}}$.

assumptions on the dynamic market game between the incumbent firms are needed to sustain the scenario. In the case of a negative feedback loop, the higher minimum share does – presumably – cause a deceleration of the Postal Service's competitive revenues, relatively to its costs and/or relatively to the rate of change in competitors' revenue. Here also, to sustain the scenario, one would need to make highly questionable assumptions on the channel through which the minimum share affects the market game. These remarks are not to say that the minimum share does not affect the market game. It simply questions any potential claim that it does so in a very specific way.

The question regarding the exhibition of possible feedback loops is closely related to the general question of the stability properties of the formula. There are diverse types of stability with corresponding mathematical definitions. Some of them are classic in system theory. I focus here on two notions of stability, using the notations that I have adopted in the declaration.

I begin by remarking that the system described by the new equation is *non-autonomous*. To explain this notion, I note that Equation (4) can, more generally, be written as

$$AS_{t+1} = f(AS_t, t) \qquad (23)$$

where $f(AS_t, t)$ denotes the expression $AS_t\{1 + r_{CCM}(t) + S(t-1)[r_{Po}(t) - r_{co}(t)]\}$.

In other words, some function, $f(.,t)$, *which depends on time,* transforms the share AS_t, set for the fiscal year *t,* into the share AS_{t+1} for the fiscal year *t+1*. The function, in the present context, depends on time through the growth rates $r(t) = r_{CCM}(t) + S(t-1)[r_{Po}(t) - r_{co}(t)$. If this growth rate is constant over time, $r(t) = r, t = 1,2,3, etc,$

then the system is called *autonomous,* in which case, the equation (23) becomes

$$AS_{t+1} = f(AS_t) \qquad (24)$$

Next, I note that the notions of stability, which will be defined below, *applies to an equilibrium point* – in our case, an equilibrium share – implied by the formula. Hence, the proper terminology is the *stability of an equilibrium share,* rather than the *stability of* *the formula*, even though I will be using these two expressions interchangeably. I now define the notion of equilibrium share.

Definition: Equilibrium Share

For the non-autonomous system $AS_{t+1} = f(AS_t, t)$, *the share* AS *is an equilibrium share if and only if* $AS = f(AS, t)$ *for all t, t=0,1,2, etc.*[29]

By simply substituting the value 0 for AS_t in equation (4) it is clear that $AS = 0$ is an equilibrium share in the equation, i.e., $0 = f(0, t)$ *for all t.* In fact, as one can verify, it also is the only equilibrium share.

I define, next, the notion of stability. In words and as already stated above, *Lyapunov* stability of an equilibrium share is about the wandering of the computed shares towards the equilibrium share. A related notion of stability is *asymptotic stability*. Asymptotic stability is about the properties of the computed shares to gradually approach the equilibrium share as time evolves. More precisely,

2.3.3. Stability[30]

The equilibrium point $AS = 0$

- is *Lyapunov-stable* (stable in the sense of *Lyapunov*) if for each $\varepsilon > 0$, and any time $t = t_0$, there is $\delta = \delta(\varepsilon, t_0)$ > 0 (that may depend on ε, and t_0) such that $AS_{t_0} < \delta(\varepsilon, t_0)$ implies $AS_t < \varepsilon$ for all t following t_0.[31]
- is *asymptotically stable* if it is *Lyapunov* stable and there is a positive constant $k(\varepsilon) > 0$ (that may depend on ε), such that AS_t converges to the equilibrium share 0 as *t* tends to infinity, provided that the inequality $AS_{t_0} < k(\varepsilon)$ holds.[32]

[29] So, once the recursion reaches an equilibrium share, it keeps reproducing that share forever.

[30] See, for example, Slotine J-J. E. , and W. Li, *Applied Nonlinear Control*, Prentice Hall, 1991, at 48-50.

[31] The last two inequalities should be stated with AS_{t_0} and AS_t taken in absolute values. In our case, however, the shares are non-negative numbers, hence equal to their absolute values.

[32] In other words, if the share at some time t_0 is set close enough to zero (that is what condition $AS_{t_0} < k(\varepsilon)$ means), then the subsequent shares converge to zero as if they fall under the spell of a force which direct them progressively towards zero.

In the equation (4), the equilibrium point $AS = 0$ is clearly *Lyapunov*-stable if the growth rate $r(t) = r_{CCM}(t) + S(t-1)[r_{Po}(t) - r_{co}(t)$ is negative for all fiscal years.[33] This sufficient condition is undesirable, however, since it means that the Postal Service's ability to pay degrades – in relative terms – every fiscal year.[34] The Commission should not have any particular preference for the zero share, compared to other non-zero shares. It may be important, in this regard, to recall the statutory provisions regarding the review of the appropriate share by the Commission:

> …the Postal Regulatory Commission shall conduct a review to determine whether the institutional costs contribution requirement under subsection (a)(3) should be retained in its current form, modified, or eliminated. 39 U.S.C. § 3633 (b).

The elimination of the institutional costs contribution requirement under subsection (a)(3) is effectively equivalent to setting the appropriate share permanently to zero. This is, however, just one of the options that the Commission is asked to consider.

A pattern of strictly decreasing rates $r(t)$ is, however, not a necessary condition for *Lyapunov* stability and the fact that it can be realized by a problematic scenario does not make this type of stability, an absolutely undesirable property. In fact, in the specific case under consideration, *Lyapunov* stability has the consequence that it implies the convergence of the shares to some limit share.

To show this, I assume that the zero-share is *Lyapunov* stable. This means that I can choose $\delta(\varepsilon, t_0)$ such that, once the inequality $AS_{t_0} < \delta(\varepsilon, t_0)$ is satisfied, all the subsequent shares – subsequent to t_0– satisfy the inequality

$AS_t < \varepsilon$ or, using equation (17), $AS_t = AS_{t_0} \prod_{j=0}^{j=t-1} \{1 + r(j)\} < \varepsilon$

The last inequality implies, however, by letting *t* grow indefinitely, that the infinite product $AS_{t_0} \prod_{j=0}^{\infty} \{1 + r(j)\}$ is finite in magnitude (because it is less than or equal to

[33] Indeed, choosing $\delta(\varepsilon, t_0) = \varepsilon$, all the AS_t, for the t following t_0, will be smaller than ε . This is so, because they are decreasing in magnitude (negative growth rate) and the first, AS_{t_0}, is less than $\delta(\varepsilon, t_0) = \varepsilon$.

[34] This is an example in which stability may not be a desirable property, for a system.

$\varepsilon)$[35] . Either one of the AS_t is zero (in which case all the following are), or none is and the shares converge to a limit that is not necessarily null, a topic that I discuss below.
To summarize, *Lyapunov* stability implies the convergence of the computed shares, but not necessarily to zero (nor to 1). In fact, the mere convergence of the share does not mean or imply asymptotic stability, which requires the limit to be equal to the zero-share.
It should be apparent from the above development that in the present context, *Lyapunov* stability cannot be unequivocally proved or disproved, because any proof would have to rely on unverifiable, hence questionable, assumptions on the dynamics of the Postal Service's ability to pay.
I now turn to the long run properties of the formula.

2.3.4. What to Expect in the Long-Run

By definition, the right-hand size of (17) converges to a non-zero limit if, beyond some time point, say τ , none of the factors vanishes and the partial products

$$p_n \coloneqq (1 + r(\tau + 1))(1 + r(\tau + 2))(1 + r(\tau + 3)) \ldots . (1 + r(\tau + n))$$

converge, as n increases, to a limit that is finite and different from 0.[36] A necessary condition for this to occur is that the growth rate $r(t)$ tends to zero as t tends to infinity.[37] As it will be become apparent below, this last condition essentially means that product-specific price-to-unit-attributable-cost and product-specific price ratios between the Postal Service and its competitors, eventually reach steady states. A simple application of a result by Knopp (1954)[38] leads to the seemingly obvious conclusion that

[35] If a sequence of real numbers is convergent and has all its terms (starting with some rank) strictly less than some positive ε, then the limit is less than *or equal to* ε. The "equal to" part of the sentence comes from the fact that the limit of the sequence has to be a limit point (or an accumulation point) of the open interval $(-\infty, \varepsilon)$. Since the boundary point, ε, is one of these limit points, it could possibly be the limit of the sequence.
[36] Cf. Konrad Knopp, *Theory and Application of Infinite Series,* Chapter VII "Infinite Products", Blackie and Son Limited, at 218.
[37] Since both p_n and p_{n-1} converge to the same non-zero limit, their ratio $\frac{p_n}{p_{n-1}} = 1 + r(n)$ converges to 1. In other words, a necessary condition for convergence is that the growth rate $r(t)$ tends to zero as t tends to infinity. Knopp. Op. Cit. at 219.
[38] Knopp Op.cit. Proof of Theorem 5. Page 222.

the computed shares converge to a limit if and only if they eventually become arbitrarily close to one another.[39]

There are various sufficient conditions for the convergence.[40] One is that the infinite sum $\sum_{\tau=1}^{\infty} r(\tau)$ be absolutely convergent, i.e., $\sum_{\tau=1}^{\infty} |r(\tau)|$ be a convergent series.[41] Another is that both $\sum_{\tau=1}^{\infty} r(\tau)$ and $\sum_{\tau=1}^{\infty} r(\tau)^2$ be convergent series. Intuitively, the last conditions have the following meaning: if one measures the size of the infinite sequence of change rates, $(r(1), r(2), r(3)\ \ldots)$ by the limit of the sum $\sum_{\tau=1}^{t} r(\tau)$, or of the sum $\sum_{\tau=1}^{t} r(\tau)^2$, then the sufficient condition is that these two sizes be finite in magnitude.

Since the limit – in case of convergence – depends on the initial share, it can, at least in principle, be controlled by properly setting this initial share. In that sense, convergence may be viewed as a desirable property. However, conditions that insure this property, such as the ones provided above, are, here also, impossible to verify directly from the formula, as they concern the entire sequence $(r(1), r(2), r(3)\ \ldots)$ of change rates, hence, the time path of the Postal Service's ability to pay.[42]

If the underlying concern for worrying about the stability of the formula is the possibility that it generates patterns of shares with abrupt and unpredictable changes, then more intuitions on the likelihood of these patterns can be gained from examining how the relative rates of change in the Postal Service's ability to pay relates to the rates of

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[39] More precisely, the computed shares converge to a limit if and only if, given $\varepsilon > 0$, along with some initial time t_0 and a positive integer k (all three arbitrary), after a suitable time following t_0, all the computed share satisfy the inequality $\left|\frac{AS_{t+k} - AS_t}{AS_t}\right| < \varepsilon$, i.e. the shares become arbitrarily close to one another.

[40] Cf, Knopp, Op. Cit. pp 222-226.

[41] Intuitively, if the size of the series of change rates in the computed shares, $r(1), r(2), \ldots$ is measured by the limit when t tends to infinity of the sum $\sum_{\tau=1}^{t} |r(\tau)|$ then the required condition is that this size be finite in magnitude**.**

[42] One may be tempted to take the logarithms of both sides of $AS_t = AS_{t_0} \prod_{j=0}^{j=t-1} \{1 + r(j)\}$ and examine the series $\sum_{j=0}^{\infty} ln\{1 + r(j)\}$ for convergence, which is not an easier exercise than its the product-form counterpart. In fact, the convergence problem becomes very difficult when $r(j)$ is considered random, because in that case, one has to state and discuss the realistic satisfaction of general conditions of convergence of a random series, such as those in Wu, W. B. and M. Woodroofe , "Martingale Approximations for Sums of Stationary Processes," The Annals of Probability, Vol. 32, No. 2 (Apr., 2004), pp. 1674-1690.

change in product-specific price-to-costs – more precisely, price to per-piece attributable cost– , and price ratios, relatively to competitors' prices. The only assumption needed here for this kind of analysis is that the Postal Service (PS) and its competitors produce (pairwise) comparable products and the total number of such pairs of products on the market is *N*.[43]

To better appreciate the conclusions that will follow below from the analytic development, it is important, I believe, to say a word about the overarching perspective. As already stated (on page 14), the reasoning underlying the new formula is to index the change in the appropriate share to the change in the Postal Service's ability to pay. This can be alternatively be expressed as the equality
[*change rate in the appropriate share =change rate in the Postal Service's ability to pay*].[44]
From this fact follows the consequence that the formula-generated sequence of appropriate shares and the sequence, $r(t), t = 1,2, etc.$, of the relative growth rates in the Postal Service's ability to pay, have *exactly* the same time-trend.[45] Hence, to predict the time-trends in the first, one needs to understand what is at play in the second, especially at product-specific level. The question to ask is, therefore, *what drives the trends in the Postal Services ability to pay at the micro-level, i.e., at product-specific level?*

The rate of change of the share in the fiscal year *t*, $r(t)$, has already been shown to satisfy the equalities (Relation (21) and (22))

$$r(t)=\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1}[r_{Po}(t)-r_{CAC}(t)]+\ S(t-1)[r_{Po}(t)-r_{co}(t)]$$

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[43] In a comparative perspective, the assumption is necessary. It is implicit and, sometime, explicit in theoretical frameworks that compare a firm's pricing behavior to its competitors'. For example, in Sappington, D. E. M., and J. G. Sidak, "Incentives for Anticompetitive Behavior by Public Enterprises," *Review of Industrial Organization* **22:** 183–206, 2003, the authors make similar assumptions about the products that are traded on the market. At 187.

[44] Or, $\frac{AS_{t+1}-AS_t}{AS_t}=r(t)$.

[45] An analogy may be useful here: if, for example, some given price is strictly indexed to inflation, then the trend in the price is nothing more or less than the trend in inflation.

or

$$r(t)=\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1} r_{\left(\frac{R_{Po}}{CAC}\right)}(t)+\ S(t-1)r_{\left(\frac{R_{Po}}{R_{co}}\right)}(t)$$

Hence, given the values of the year *t-1* factors, $\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1}$ and $S(t-1)$,[46] the rates $r(t)$ is a combination of the two rates $r_{\left(\frac{R_{Po}}{CAC}\right)}(t)$ and $r_{\left(\frac{R_{Po}}{R_{co}}\right)}(t)$. Switching the focus to the two underlying ratio, $\frac{R_{Po}}{CAC}$ and $\frac{R_{Po}}{R_{co}}$, I show in the appendix the equalities

$$\frac{R_{Po}(t)}{CAC(t)}=\sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)\left(\frac{CAC_i(t)}{\sum_{i=1}^{i=N}CAC_i(t)}\right) \quad (25)$$

$$=\sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)w_{a,i}(t), \text{where } w_{a,i}(t):=\left(\frac{R_{co,i}(t)}{\sum_{i=1}^{i=N}R_{co,i}(t)}\right), \quad (26)$$

Where $a_i(t)$ denotes, I recall, the unit attributable cost.

Hence, the revenue-to-attributable-cost ratio, $\frac{R_{Po}(t)}{CAC(t)}$, is expressible as a weighted average of the product-specific price-to-cost ratios, $\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)$, with the corresponding weights $w_{a,i}(t):=\left(\frac{CAC_i(t)}{\sum_{i=1}^{i=N}CAC_i(t)}\right)$. Since the terms $\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)w_{a,i}(t)$ are all non-negative, the growth rate of the sum is a weighted average –with weights that are different from $w_{a,i}(t)$ – of the growth rates of these terms.[47] As an average, it is certainly not larger than the maximum of the growth rates of the terms. One may then ask the following question :

Question 1

What time-path are the product-specific price-to-cost ratios likely to follow over a given period of time, and what time-path are their corresponding cost weights, $w_{a,i}(t)=\frac{CAC_i(t)}{\sum_{i=1}^{i=N}CAC_i(t)}$*, likely to follow over the same period of time? Are these paths likely to be moderate or, instead, chaotic, explosive, unpredictable?*

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[46] The ratio $\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1}$ can also be written as $\left(\frac{1-CCM}{CCM}\right)_{t-1}$. The other factor, $S(t-1)$ is, I recall, the Postal Service's market share in the fiscal year *t-1.*

[47] The growth rate of $\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)w_{a,i}(t)$ is the sum of the growth rates of the factors.

The second ratio, $\frac{R_{Po}}{R_{co}}$, can similarly be decomposed as the first to obtain

$$\frac{R_{Po}}{R_{co}} = \sum_{i=1}^{i=N} \left(\frac{p_{Po,i}(t)}{p_{co,i}(t)}\right) w_{co,i}(t), \text{where } w_{co,i}(t) := \left(\frac{R_{Po,i}(t)}{\sum_{i=1}^{i=N} R_{Po,i}(t)}\right) \quad (27)$$

In other words, the ratio $\frac{R_{Po}}{R_{co}}$ is the average of the product-specific price ratios, $\frac{p_{Po,i}(t)}{p_{co,i}(t)}$, weighted by the competitors' revenue-weights $w_{co,i}(t) := \left(\frac{R_{Po,i}(t)}{\sum_{i=1}^{i=N} R_{Po,i}(t)}\right)$. The second question of interest is, therefore:

Question 2

What time-path are the product-specific price-ratios, $\frac{p_{Po,i}(t)}{p_{co,i}(t)}$*, likely to follow over a given period of time, and what time-path are their corresponding revenue weights,* $w_{co,i}(t) := \left(\frac{R_{Po,i}(t)}{\sum_{i=1}^{i=N} R_{Po,i}(t)}\right)$*, likely to follow over the same period of time? Are these paths likely to be moderate or, instead, chaotic, explosive, unpredictable?*

Answers to and Implications of Questions 1 and 2

If the variables in questions 1 and 2 are expected to remain on moderate paths over the period of interest, so will be the shares computed by the formula. If they are expected to be rather volatile, so will again be the shares computed by the formula.[48]

From a predictive perspective, the stability of the formula (the word is used in this sentence to refer to an unpredictable, erratic behavior) depends, not on market equilibrium outcomes – price-to-unit-cost ratios and price ratios relative to competitors, or cost and revenue structures –, if any, as they can be predicted by alternative economic theories or models, but rather on whether these predicted outcomes are expected to be changing erratically from year to year. Hence, a theory that assumes or predicts that the equilibrium outcomes, if any, do not change drastically over time does, also predicts, in fact, that the formula will be stable, in the sense that it will produce shares with moderate time paths. This is so because the formula is only about the rates of change over time in these outcomes. It is not about the levels of these outcomes.

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[48] In fact, the formation of expectations about the time path of the Postal Service's ability to pay is even more difficult if one considers the possibility that this path is affected by possible feedback loops.

The ratio nature of these outcomes is also very important. In fact, if one asks

W*hat does the formula produce if all revenues,* $R_{Po}(t)$ *and* $R_{co}(t)$ *, and the attributable cost* $CAC(t)$ *grow over time at a same rate?*

The answer is readily provided by the equality

$$r(t)=\left(\frac{CAC}{R_{Po}-CAC}\right)_{t-1}[r_{Po}(t)-r_{CAC}(t)]+S(t-1)[r_{Po}(t)-r_{co}(t)].$$

If $\left(r_{Po}(t)=r_{CAC}(t)=r_{co}(t)\right)$, then $r(t)=0$, in other words, the computed share remains unchanged over time under the stated assumption, a fortiori if that common growth rate is constant over time (steady state equilibrium). It should not come as a surprise, therefore, that, provided the two revenues, $R_{Po}(t)$ and $R_{co}(t)$, and the attributable cost, $CAC(t)$, all change at the same annual rate, say $\gamma(t)$, and regardless of how fluctuating the rate $\gamma(t)$ may be from one year to the next, the computed appropriate share will remain unchanged over the relevant period of time. This is so, once again, because only the growth rates of the ratios $R_{Po}(t)/R_{co}(t)$, and $CCA(t)/R_{Po}(t)$ drive the updating mechanism built into the formula. Not the individual growth rates of $R_{Po}(t)$, $R_{co}(t)$, and $CAC(t)$.

To conclude about the topic of stability, I will state the following: the *Lyapunov*-stability of the equilibrium share may be a desirable property, because it implies the long-term convergence of the computed shares to a limit, which depends on the magnitude of the initial share. However, as stated above, as it pertains to the Commission's formula, *Lyapunov*-stability, and a fortiori, asymptotic stability, cannot be proved or disproved without making questionable assumptions on the dynamics of the Postal Service's ability to pay.

As for the odds of erratic trends in the time paths of the computed shares, they ultimately depend on whether the fundamental equilibria, if any, associated with the underlying market games, which produces the observable trends in prices and volumes, both at product and aggregate levels, have moderate time evolutions or not.

Because, in the present case, any formal proof of the discussed types of stability – or of the lack of stability – cannot avoid relying on unverifiable assumptions on the likely time path of the Postal Service's ability to pay, an alternative – and more modest – goal is to provide in the appendix, a piece of simulation-based information, with the hope that it will help gain some insights about the time paths of the calculated shares over relatively short horizons. The discussion of this simulation strategy is the purpose of the following section.

2.3.5. Scenarios for a Five-Year Period

Equation (4) is reproduced here for convenience:

$$AS_{t+1} = AS_0 \prod_{j=0}^{j=t}\{1 + r(j)\} \text{ ,[49]} \qquad (17)$$

The time paths of the minimum shares produced by the new formula for the next five years can be simulated based on values assigned to the growth rate, $(r_{CCM}(t) + S(t-1)[r_{Po}(t) - r_{co}(t)])$. The historically highest rate computed from the data provided along with Order No. 4742, occurred over the time span FY2016 to FY2017 and is equal to +19.5%. In the setting of the new formula, this corresponds to the largest annual relative increase in the Postal Service's ability to pay over the period 2007-2017. The largest relative decrease in its ability to pay was -8.1% and did occur over the time span FY2016 to FY2017. Using these extreme rates of change, conservative paths can be determined for the appropriate share over the next 5-year period. There are 32 different paths that can be derived from this assumption of binary annual changes and they are described in Table 1 in the appendix. In the table, the "plus" sign denotes an increase while the "minus" sign, a decrease. Each of these paths takes as the initial share, the share computed by the Commission for the fiscal year 2018, which is 8.8%. Table 2 in the appendix repeats the simulation with the initial share set to its current level, 5.5%.

[49] This form is obtained by repeated substitution of $AS_{j-1}\{1 + r(j-1)\}$ for AS_j on the right-hand side of Equation (4), until j is equal to 1.

The two most extreme paths are the monotone – strictly increasing or decreasing – paths, which are Path 1 and Path 32. All the other paths are, each, fluctuating to some degree.

To summarize, the scenarios – in Table 1– would generate paths of shares that are bounded below, *uniformly* – i.e., all shares in the path are bounded below – by 5.8% and bounded above, *again uniformly*, by 21.4%.

This simulation is based, it is important to recall this fact, on simple extrapolations of the extreme figures observed over the period 2007-2017. The actual future figures may be even larger in some years than assumed in my scenarios. While 21.4%, the highest 5th-year share – achieved in Path 1 – may seem excessive as a minimum, one should remember that it corresponds to the historically highest relative growth rate of the Postal Service's ability to pay, as measured by the formula.

3. Conclusion

Three conclusions can be derived from this assessment:

1. The indexation by the formula of the change in the appropriate share to the change in the Postal Service's ability to pay is a sound strategy. However, if this strategy is translated into a decision rule, it raises a question about the principle of equalization between the Postal Service's ability to pay and its obligation to pay.

2. *Lyapunov* stability, *asymptotic* stability, or simple convergence of the calculated shares cannot be formally proved or disproved without relying on questionable assumptions on the time path followed by the Postal Service's ability pay.

3. The likelihood of unpredictable erratic trends in the calculated appropriate shares is equal to that of identical trends in the Postal Service's ability to pay relative to the referential baselines. This is a simple consequence of the indexation rule described in Point 1.

Appendix

I. Simulation Results[50]

[50] The shares are for the following year.

Table 1: Share Paths Based on Historically Extreme Rates of Change. Initial Rate: 8.8%

FYear / Path	1	2	3	4	5	1	2	3	4	5
1	+	+	+	+	+	10.52%	12.57%	15.02%	17.95%	21.44%
2	+	+	+	+	−	10.52%	12.57%	15.02%	17.95%	16.49%
3	+	+	+	−	+	10.52%	12.57%	15.02%	13.80%	16.49%
4	+	+	+	−	−	10.52%	12.57%	15.02%	13.80%	12.68%
5	+	+	−	+	+	10.52%	12.57%	11.55%	13.80%	16.49%
6	+	+	−	+	−	10.52%	12.57%	11.55%	13.80%	12.68%
7	+	+	−	−	+	10.52%	12.57%	11.55%	10.61%	12.68%
8	+	+	−	−	−	10.52%	12.57%	11.55%	10.61%	9.75%
9	+	−	+	+	+	10.52%	9.66%	11.55%	13.80%	16.49%
10	+	−	+	+	−	10.52%	9.66%	11.55%	13.80%	12.68%
11	+	−	+	−	+	10.52%	9.66%	11.55%	10.61%	12.68%
12	+	−	+	−	−	10.52%	9.66%	11.55%	10.61%	9.75%
13	+	−	−	+	+	10.52%	9.66%	8.88%	10.61%	12.68%
14	+	−	−	+	−	10.52%	9.66%	8.88%	10.61%	9.75%
15	+	−	−	−	+	10.52%	9.66%	8.88%	8.16%	9.75%
16	+	−	−	−	−	10.52%	9.66%	8.88%	8.16%	7.50%
17	−	+	+	+	+	8.09%	9.66%	11.55%	13.80%	16.49%
18	−	+	+	+	−	8.09%	9.66%	11.55%	13.80%	12.68%
19	−	+	+	−	+	8.09%	9.66%	11.55%	10.61%	12.68%
20	−	+	+	−	−	8.09%	9.66%	11.55%	10.61%	9.75%
21	−	+	−	+	+	8.09%	9.66%	8.88%	10.61%	12.68%
22	−	+	−	+	−	8.09%	9.66%	8.88%	10.61%	9.75%
23	−	+	−	−	+	8.09%	9.66%	8.88%	8.16%	9.75%
24	−	+	−	−	−	8.09%	9.66%	8.88%	8.16%	7.50%
25	−	−	+	+	+	8.09%	7.43%	8.88%	10.61%	12.68%
26	−	−	+	+	−	8.09%	7.43%	8.88%	10.61%	9.75%
27	−	−	+	−	+	8.09%	7.43%	8.88%	8.16%	9.75%
28	−	−	+	−	−	8.09%	7.43%	8.88%	8.16%	7.50%
29	−	−	−	+	+	8.09%	7.43%	6.83%	8.16%	9.75%
30	−	−	−	+	−	8.09%	7.43%	6.83%	8.16%	7.50%
31	−	−	−	−	+	8.09%	7.43%	6.83%	6.28%	7.50%
32	−	−	−	−	−	8.09%	7.43%	6.83%	6.28%	5.77%

Table 2: Share Paths Based on Historically Extreme Rates of Change. Initial rate: 5.5%

Path \ FYear	1	2	3	4	5	1	2	3	4	5
1	+	+	+	+	+	6.57%	7.85%	9.39%	11.22%	13.40%
2	+	+	+	+	−	6.57%	7.85%	9.39%	11.22%	10.31%
3	+	+	+	−	+	6.57%	7.85%	9.39%	8.63%	10.31%
4	+	+	+	−	−	6.57%	7.85%	9.39%	8.63%	7.93%
5	+	+	−	+	+	6.57%	7.85%	7.22%	8.63%	10.31%
6	+	+	−	+	−	6.57%	7.85%	7.22%	8.63%	7.93%
7	+	+	−	−	+	6.57%	7.85%	7.22%	6.63%	7.93%
8	+	+	−	−	−	6.57%	7.85%	7.22%	6.63%	6.10%
9	+	−	+	+	+	6.57%	6.04%	7.22%	8.63%	10.31%
10	+	−	+	+	−	6.57%	6.04%	7.22%	8.63%	7.93%
11	+	−	+	−	+	6.57%	6.04%	7.22%	6.63%	7.93%
12	+	−	+	−	−	6.57%	6.04%	7.22%	6.63%	6.10%
13	+	−	−	+	+	6.57%	6.04%	5.55%	6.63%	7.93%
14	+	−	−	+	−	6.57%	6.04%	5.55%	6.63%	6.10%
15	+	−	−	−	+	6.57%	6.04%	5.55%	5.10%	6.10%
16	+	−	−	−	−	6.57%	6.04%	5.55%	5.10%	4.69%
17	−	+	+	+	+	5.05%	6.04%	7.22%	8.63%	10.31%
18	−	+	+	+	−	5.05%	6.04%	7.22%	8.63%	7.93%
19	−	+	+	−	+	5.05%	6.04%	7.22%	6.63%	7.93%
20	−	+	+	−	−	5.05%	6.04%	7.22%	6.63%	6.10%
21	−	+	−	+	+	5.05%	6.04%	5.55%	6.63%	7.93%
22	−	+	−	+	−	5.05%	6.04%	5.55%	6.63%	6.10%
23	−	+	−	−	+	5.05%	6.04%	5.55%	5.10%	6.10%
24	−	+	−	−	−	5.05%	6.04%	5.55%	5.10%	4.69%
25	−	−	+	+	+	5.05%	4.65%	5.55%	6.63%	7.93%
26	−	−	+	+	−	5.05%	4.65%	5.55%	6.63%	6.10%
27	−	−	+	−	+	5.05%	4.65%	5.55%	5.10%	6.10%
28	−	−	+	−	−	5.05%	4.65%	5.55%	5.10%	4.69%
29	−	−	−	+	+	5.05%	4.65%	4.27%	5.10%	6.10%
30	−	−	−	+	−	5.05%	4.65%	4.27%	5.10%	4.69%
31	−	−	−	−	+	5.05%	4.65%	4.27%	3.92%	4.69%
32	−	−	−	−	−	5.05%	4.65%	4.27%	3.92%	3.61%

II. Effect of Changing the type of mean used to compute the industry average revenue

Table 3: Change in CGD induced by a change in the type of mean

Type of Mean	Formula	Rate of change in the Appropriate share
Harmonic	$\left[\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\right]^{-1}$, *a, b >0*	$r(t) = 1 + r_{CCM}(t) + \boldsymbol{S(t-1)}[r_{Po}(t) - r_{co}(t)]$
Arithmetic	$\frac{1}{2}(a+b)$	$r(t) = 1 + r_{CCM}(t) + (\boldsymbol{1 - S(t-1)})[r_{Po}(t) - r_{co}(t)]$
Geometric	$\sqrt{ab}$, *a, b >0*	$r(t) = 1 + r_{CCM}(t) + \boldsymbol{\frac{1}{2}}[r_{Po}(t) - r_{co}(t)]$

As Table 3 shows, the induced change is in the factor multiplying the difference,

$[r_{Po}(t) - r_{co}(t)]$, between the two growth rates. This factor is fixed to half when the geometric mean is used. Using the arithmetic mean - the usual average – has the effect of changing the factor from the Postal Service's market share to its competitors' market share.[51]

III. Proofs of Inequalities (15) and (16)

I first note

$$\left(1 + \frac{R_{Po}(t)}{R_{co}(t)}\right) = \left(\frac{R_{co}(t)}{R_{Po}(t) + R_{co}(t)}\right)^{-1} = \left(\frac{1}{1 - S(t)}\right),$$

which directly implies (15). Further,

$$\left(\frac{R_{Po}(t) - CAC(t)}{R_{Po}(t)}\right)\left(1 + \frac{R_{Po}(t)}{R_{co}(t)}\right) = (R_{Po}(t) - CAC(t))\left(\frac{R_{co}(t) + R_{co}(t)}{R_{Po}(t)R_{co}(t)}\right) =$$

$$= (R_{Po}(t) - CAC(t))\left(\frac{1}{R_{Po}(t)} + \frac{1}{R_{co}(t)}\right) = \frac{R_{Po}(t) - CAC(t)}{\frac{1}{2}\left(\frac{1}{2}\left[\frac{1}{R_{Po}(t)} + \frac{1}{R_{co}(t)}\right]\right)^{-1}}$$

$$= \frac{R_{Po}(t) - CAC(t)}{\frac{1}{2} H(R_{Po}(t); R_{co}(t))} = J_2,$$

as claimed.

IV. Proof of the Equality $\frac{R_{Po}(t)}{CAC(t)} = \sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)\left(\frac{CAC_i(t)}{\sum_{i=1}^{i=N} CAC_i(t)}\right)$

$$\frac{R_{Po}(t)}{CAC(t)} = \frac{\sum_{i=1}^{i=N} R_{Po,i}(t)}{\sum_{i=1}^{i=N} CAC_i(t)} = \frac{\sum_{i=1}^{i=N} p_{Po,i}(t)Q_{Po,i}(t)}{\sum_{i=1}^{i=N}\left(\frac{CAC_i(t)}{Q_{Po,i}(t)}\right)Q_{Po,i}(t)} = \frac{\sum_{i=1}^{i=N} p_{Po,i}(t)Q_{Po,i}(t)}{\sum_{i=1}^{i=N}\left(\frac{CAC_i(t)}{Q_{Po,i}(t)}\right)Q_{Po,i}(t)} = \frac{\sum_{i=1}^{i=N} p_{Po,i}(t)Q_{Po,i}(t)}{\sum_{i=1}^{i=N} a_i(t)Q_{Po,i}(t)} \quad (28)$$

$$= \frac{\sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)a_i(t)Q_{Po,i}(t)}{\sum_{i=1}^{i=N} a_i(t)Q_{Po,i}(t)} = \frac{\sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)CAC_i(t)}{\sum_{i=1}^{i=N} CAC_i(t)} = \sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)\left(\frac{CAC_i(t)}{\sum_{i=1}^{i=N} CAC_i(t)}\right) = \sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right)\left(\frac{CAC_i(t)}{\sum_{i=1}^{i=N} CAC_i(t)}\right) \quad (29)$$

$$= \sum_{i=1}^{i=N}\left(\frac{p_{Po,i}(t)}{a_i(t)}\right) w_{a,i}(t), \text{where } w_{a,i}(t) := \left(\frac{R_{co,i}(t)}{\sum_{i=1}^{i=N} R_{co,i}(t)}\right) \quad (30)$$

V. Handling inflation in the new formula

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[51] The proofs of the expressions in column 3 follow the same paths as on page 12.

In a continuous-time analysis, where the time runs over an interval, say [a, b], with a>0, and a<b, and the rate of change in a variable X is defined as the logarithmic derivative $r_X(t) = \frac{\frac{\partial(X)}{\partial(t)}}{X(t)} := \frac{\dot{X}}{X}$, one has from elementary properties of the derivative,

$$\frac{\dot{XY}}{XY} = \frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} \text{ and } \frac{\dot{X/Y}}{X/Y} = \frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} \qquad (31)$$

If X is positive, then the expression $\frac{\frac{\partial(X)}{\partial(t)}}{X(t)}$ can also be written as $\frac{\partial ln(X)}{\partial(t)}$.

Letting $X(t)$ and $Y(t)$ denote two revenues expressed in nominal terms, and $I(t)$ denote the price index, the corresponding deflated revenues are $\frac{X(t)}{I(t)}$ and $\frac{Y(t)}{I(t)}$. So, the rates of change in the real revenues are $\frac{\dot{X}}{X} - \frac{\dot{I}}{I}$=$r_X(t) - r_I(t)$ and $\frac{\dot{Y}}{Y} - \frac{\dot{I}}{I}$=$r_X(t) - r_I(t)$. The rate of change in the ratio of the nominal revenues, $\frac{X(t)}{Y(t)}$, is, therefore,

$r_X(t) - r_Y(t) = (r_X(t) - r_I(t)) - (r_X(t) - r_I(t))$, i.e., it is equal to the rate of change in the ratio of the real revenues. Inflation (the rate of change in the price index) does not affect the differential growth rate.

In the case of discrete-time variables, the same result holds, if derivative $\frac{\partial ln(X)}{\partial(t)}$ is approximated as $\ln(X(t)) - \ln(X(t-1)) = ln\left(\frac{X(t)}{X(t-1)}\right)$.[52] Indeed, the resulting growth rate of the real revenue, for *X*, is $r_{Xreal}(t) = \ln\left(\frac{X(t)}{I(t)}\right) - \ln\left(\frac{X(t-1)}{I(t-1)}\right)$

$= [\ln(X(t)) - \ln(I(t))] - [\ln(X(t-1)) - \ln(I(t-1))] = ln\frac{X(t)}{X(t-1)} - ln\frac{I(t)}{I(t-1)}$.

Hence,

$$r_{Xreal}(t) - r_{Yreal}(t) = \left[ln\frac{X(t)}{X(t-1)} - ln\frac{I(t)}{I(t-1)}\right] - \left[ln\frac{Y(t)}{Y(t-1)} - ln\frac{I(t)}{I(t-1)}\right] = ln\frac{X(t)}{X(t-1)} - ln\frac{Y(t)}{Y(t-1)}$$

$$= r_{Xnominal}(t) - r_{Ynominal}(t) \qquad (32)$$

---

[52] For the sake of transparency, I must note that, because the general inequality $\ln(x) \leq x - 1$, holds for any positive real number x, one has $ln\left(\frac{X(t)}{X(t-1)}\right) \leq \frac{X(t)}{X(t-1)} - 1$. It follows from the last inequality that the proposed approximation to the growth rate , in fact, under-estimates $r_X(t) = \frac{X(t)}{X(t-1)} - 1$ and the closer $\frac{X(t)}{X(t-1)}$ is to 1, the better is the approximation, i.e., the lower is the approximation error. However, the closeness of $\frac{X(t)}{X(t-1)}$

A consequence of the above development is that there is no need for adjusting the revenue data for inflation if the rate

$r(t) = r_{CCM}(t) + S(t-1)[r_{Po}(t) - r_{co}(t)]$ is calculated as

$\ln(CCM(t)) - \ln(CCM(t-1)) + S(t-1)\{(\ln(R_{Po}(t)) - \ln(R_{Po}(t-1))) - (\ln(R_{co}(t)) - \ln(R_{co}(t-1)))\}$

$$= ln\left(\frac{CCM(t)}{CCM(t-1)}\right) + S(t-1)\left[ln\left(\frac{R_{Po}(t)}{R_{Po}(t-1)}\right) - ln\left(\frac{R_{co}(t)}{R_{co}(t-1)}\right)\right] := \tilde{r}(t) \qquad (33)$$

Using (33), I calculated the corresponding shares using the Commission's initial share.[53] The results are compared in the Table 4 and Figure 1.

Table 4: Comparing the formula-(33) based shares to the Order No. 4742 shares

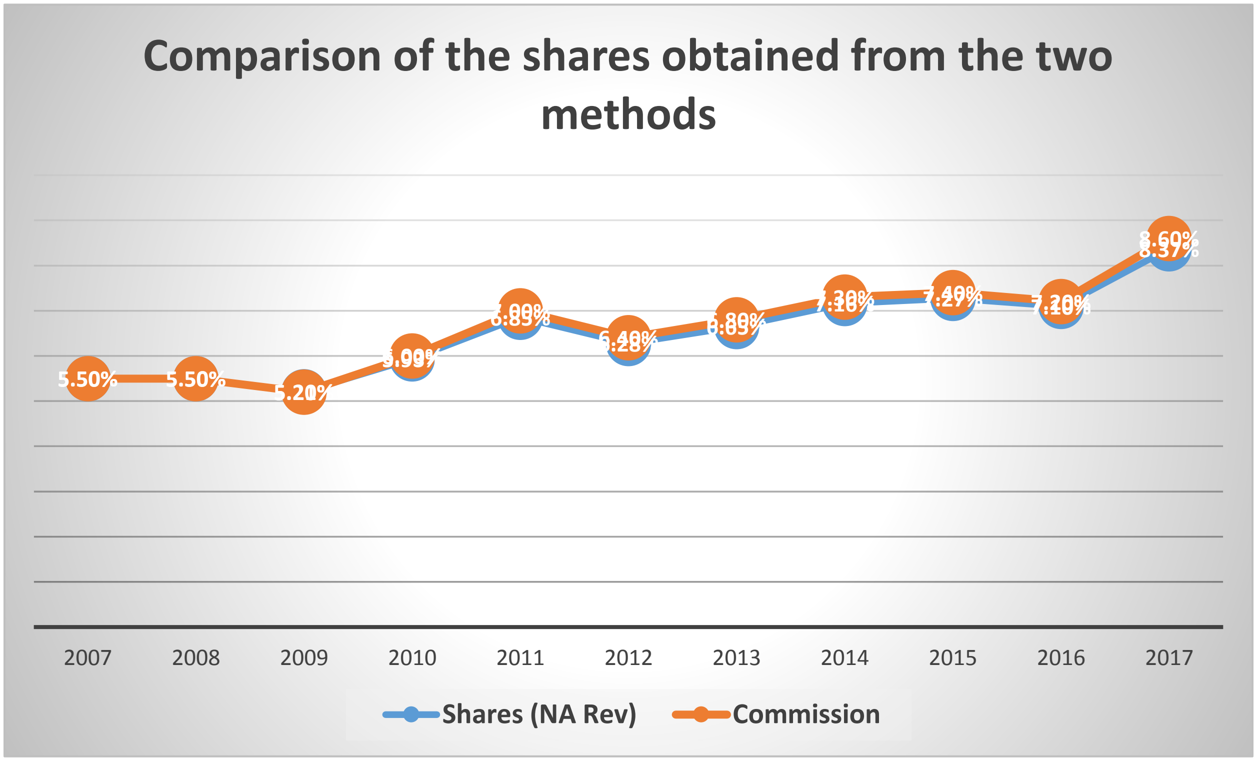
Shares (NA Rev) stands for "Shares based on a non-adjustment of revenues for inflation."

Fiscal Year	Shares (NA Rev)	Commission
2007	5.50%	5.50%
2008	5.50%	5.50%
2009	5.21%	5.20%
2010	5.93%	6.00%
2011	6.85%	7.00%
2012	6.28%	6.40%
2013	6.65%	6.80%
2014	7.16%	7.30%
2015	7.27%	7.40%
2016	7.10%	7.20%
2017	8.37%	8.60%

to 1 may be viewed as an implicit assumption that is made on the magnitude of the inflation rate.

[53] To be consistent with the equality $r(t) = r_{J_0}(t)$ (see page 12, relation (14)), the CGD should simply be calculated as $ln\left(\frac{1+\frac{R_{Po}(t)}{R_{Co}(t)}}{1+\frac{R_{Po}(t-1)}{R_{Co}(t-1)}}\right)$. Using this alternative method does not change the results compared to the one obtained from (33).

Figure 1: Comparing the formula-(33) based shares to the Order 4742 shares



VERIFICATION

I SOILIOU DAW NAMORO, declare under penalty of perjury that the foregoing 's true and correct to the best of my knowledge. Executed on September 12, 2018.

Soiliou Daw Namoro